Rational Attitude Change by Reference Cues when Information Elaboration Requires Effort

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Abstract

In this paper we develop a parsimonious model of decision-making that aims at capturing the determinants of consumers' attitude change in response to persuasive messages that exploit reference cues. Our model is inspired by dual-process theories of information elaboration, but does not introduce purely behavioral traits. The decision-maker receives a message containing an offer of unknown quality together with a reference cue that associates the offer with a category of offers, whose average quality is known. The decision-maker initially exerts little cognitive effort in processing the message, assessing the offer quality on the sole basis of the reference cue (acting as a "coarse thinker"). The decision-maker can then take a decision on the offer without further investigating or she can exert substantial cognitive effort to scrutinize the message carefully and obtain more precise knowledge of the offer quality. The proposed model predicts that the persuader can exploit reference cues to affect attitudes both directly (by inducing acceptance of offers) and indirectly (by inducing low cognitive effort, and hence influencing acceptance). This model matches several predictions of prominent psychological models of attitude change such as the Elaboration Likelihood Model and the Heuristic-Systematic Model.

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1 Introduction

Consider the following situation. A milk producer can bottle milk in either glass or plastic. Glass is more expensive than plastic, and has no impact on milk quality per se. A consumer has to decide whether to buy a bottle of milk, and can choose whether to exert high or low effort in evaluating the quality of the milk offered. If the consumer exerts low effort, then she only relies on the fact that milk is contained in a glass bottle, and she generically thinks of products in glass containers, whose average quality can be higher than that of products in plastic containers. If the consumer exerts high effort, then she reads and understands labels, recovers data from memory, and elaborates all the information extracted; this is a costly activity at the end of which she is able to fully assess the quality of the product. If the milk producer anticipates the behavior of the consumer, then he can use the container – which we call *cue* (or signal) throughout the rest of the paper – to persuade the consumer to buy his product.

In this paper we develop a parsimonious model of persuasion by means of reference cues that aims at capturing the situation described above and that, at the same time, matches several predictions of prominent psychological models of attitude change such as the Elaboration Likelihood Model (Petty and Cacioppo, 1986a) and the Heuristic-Systematic Model (Eagly and Chaiken, 1993). In particular, we nest the insights from psychology about dual process information processing (high and low elaboration effort) and categorical thinking (expected quality is average quality in a category) on a game-theoretic model of communication. This allows us to rationalize the behavior of a principal who – in order to persuade an agent to act in his interest – resorts to costly communicative tools – such as the glass bottle – that are otherwise useless in a model where the agent has unbounded cognitive resources.

More precisely, we frame persuasion activities within a sender-receiver model – the sender is the *persuader*, the receiver is the *decision-maker* – assuming that agents are both rational and Bayesian. In addition, we introduce cognitive limitations along the lines suggested by social and cognitive psychology with regard to how the decision-maker can elaborate information and how the persuader can take advantage of this. In particular, as in Dewatripont and Tirole (2005), we posit that the decision-maker has to pay a cognitive cost to process information fully and precisely.[1,2] This kind of assumption is not new in models of

[1]Other recent contributions considering the costly acquisition of information are Dewatripont (2006), Caillaud and Tirole (2007), Tirole (2009) and Butler et al. (2013).

[2]Brocas and Carrillo (2008) and Brocas (2012) stress that the evidence provided by brain sciences on the multi-system nature of the human brain should be a fundamental source of inspiration for the modeling of

advertising, although with a different interpretation: the cost incurred by consumers is not cognitive in nature, rather it measures the search effort to acquire information on products' characteristics (see, e.g., Gardete and Guo, 2014).

In the modeling of cues we depart from what done in economics so far (Dewatripont and Tirole, 2005). Reference cues are modelled as the part of the message sent by the persuader that refer the object offered to a category of objects. This is particularly relevant since the decision-maker can be a "coarse thinker" (Mullainathan, 2002), i.e., she might be unable to distinguish objects falling in the same category.[3] More precisely, we assume that the decision-maker suffers from coarse thinking whenever she chooses not to bear the cognitive cost of elaborating carefully the message sent by the persuader. Thanks to this we can account for the strategic use of cues and match many findings of psychological theories of persuasion. These findings are in line with the marketing literature that shows how cues are used to influence consumers' perception of quality (see, e.g., Iyer and Kuksov, 2010), pointing out that very effective cues might have little to do with actual quality of products (Teas and Agarwal, 2000). Indeed, experimental and survey evidence indicates that these cues can exert a sizeable impact on consumers' valuation (see, e.g., Sáenz-Navajas et al., 2013; Woodside, 2012, for wine and food products, respectively). Our model can be seen as a way to rationalize the use of such cues in the attempt to persuade consumers to buy (see Bagwell, 2007, for detailed references on persuasive advertisement). Also, our model can account for persuasion activities such as the one considered in the field experiment by Bertrand et al. (2010) – where prospective borrowers are offered loans by mailed advertisement cards with different cues – which can hardly fit the model proposed by Dewatripont and Tirole (2005) since the latter only considers cues related to the sender's expertise.

We briefly sketch the basic working of our model. The persuader makes an offer to the decision-maker, and provides a reference cue – like the packaging of a product – that refers the offer to a category of offers from which the decision-maker can obtain summary information about the expected quality. The decision-maker initially processes all information under *low* elaboration, which leads her to form beliefs on the sole basis of the reference cue. Such elaboration is *automatic*, in the sense that it cannot be avoided by the decision-maker. However, the decision-maker can, at her choice, decide to exert substantial cognitive effort in order to acquire specific information on the offer quality. In this way information is

decision-making (see Brocas and Carrillo, 2014, for a focused survey).

[3]Other papers considering coarse thinking or thinking by categories are: Mullainathan et al. (2008) on persuasion; Mohlin (2014), Peski (2011) and Fryer and Jackson (2008) on optimal categorization, Ettinger and Jehiel (2010) on coarse understanding of opponents' play, and Mengel (2012) on the evolution of coarse categorization.

processed under *high* elaboration which is a *deliberate* and *controlled* cognitive process.

Importantly, we do not treat low elaboration as a behavioral shortcut, but as ignorance of which the decision-maker is aware and that she is able to quantify, so that she can form expectations and choose the elaboration level that maximizes expected utility. Coarse thinking crucially comes in to generate a tradeoff: low elaboration leads to take a decision that is good on average, but that is not necessarily the best choice for the current offer; however, low elaboration allows to save on cognitive effort, and hence the decision-maker can well decide not to engage in high elaboration, especially when beliefs about the quality of the current offer are quite extreme (in which case the decision-maker does not expect to learn much from high elaboration). Since the persuader anticipates all this, he can make a strategic use of the reference cue.[4]

Despite its simplicity, this model and its extensions prove able to predict behavior that is in line with well established findings on persuasion in the psychological literature (Petty and Cacioppo, 1986a; Eagly and Chaiken, 1993). The persuasive message sent by the persuader is interpreted by the decision-maker differently under high elaboration and low elaboration (Remark 1). A greater motivation or a better cognitive ability by the decision-maker makes the recourse to high elaboration more likely (Remarks 2 and 3). The use of reference cues by the persuader affects the elaboration effort and the reaction to the offer by the decision-maker (Remark 4). Persuasion obtained under high elaboration by the decision-maker is stabler than if obtained under low elaboration (Remark 5). Antecedents of elaboration such as arousal or prior knowledge can influence the intensity of elaboration and induce biased elaboration (Remarks 6 and 7).

The paper is organized as follows. Section 2 surveys the relevant literature on persuasion, distinguishing between contributions from economics, psychology, and marketing, as well as some recent papers that in different ways attempt to model the rational allocation of scarce cognitive resources. Section 3 presents the model in three steps: the processing of the message (Subsection 3.1), the optimal behavior of the decision-maker (Subsection 3.2), and the optimal behavior of the persuader (Subsection 3.3). Section 5 discusses the behavioral implications of the model and their consistency with psychological findings. Section 6 provides a discussion of the potential applications of the model to persuasion activities, with various examples drawn from economic, political, and social spheres. Section 7 extends the model to encompass the case of a continuum of elaboration intensities, showing that the quality of

[4] A similar mechanism drives the results in Guo and Zhang (2012), where a single firm that offers products of different quality can choose quality dispersion and prices in order to induce, or prevent, deliberation, which is needed for a consumer to unveil her own valuation of the product (not objective quality).

the model predictions remains unchanged. Section 8 concludes, summarizing the contribution and showing lines for future research. Finally, the Appendix collects proofs and related technical details (Appendix A), the exact conditions for all kinds of persuasion equilibria (Appendix B), examples and results on equilibrium existence and uniqueness (Appendix C), and some further model extensions (Appendix D).

2 Related literature

In this section we review the main contributions on persuasion, distinguishing between economic, marketing, and psychological literature. We focus mainly on theory.[5] We also briefly review recent related papers that focus on scarce cognitive resources, stressing similarities and differences with our model.

Economics. In recent years several models have been proposed that study how a message can persuade a decision-maker. We can distinguish among them on the basis of different criteria.

A first criterion is whether the act of persuasion is belief-based or non-belief-based – i.e., preference-based. Non-belief based persuasion affects behavior independently of beliefs. In such a case persuasion is obtained because the message itself impacts on utility and, hence, on behavior (Stigler and Becker, 1977; Becker and Murphy, 1993). This is also reminiscent of models of persuasive advertising (Braithwaite, 1928). Instead, belief-based persuasion affects behavior by changing decision-maker's beliefs. For instance, a decision-maker can be persuaded by informative communication (Stigler, 1961; Telser, 1964). As an application of this, models of informative advertising (see, e.g., Bagwell and Ramey, 1993) consider advertising as a way to communicate goods' characteristics to otherwise uninformed consumers.

Among the models where persuasion is understood as belief-based, a further distinction can be made between models where agents are perfect Bayesian updaters – as the model presented in this paper – and models where they are not. When agents are Bayesian updaters, a persuader, given a communication technology, chooses signals that can be costly (Nelson, 1970) or not (Gentzkow and Shapiro, 2006; Kamenica and Gentzkow, 2011) and that are correctly elaborated. Instead, non-fully Bayesian agents have limitations in the way they

[5]For a recent survey of empirical evidence regarding persuasion activities, especially related to economics and politics, see Della Vigna and Gentzkow (2010). For one regarding marketing and psychology see Dillard and Pfau (2002).

elaborate the signal: e.g., they are constrained by limited memory (Mullainathan, 2002; Shapiro, 2006), they double-count repeated information (De Marzo et al., 2003), they neglect the incentives of the sender (Eyster and Rabin, 2010), or they have limitations in the ability to lie (Glazer and Rubinstein, 2012). Both types of agents can co-exist in the same model: Gabaix and Laibson (2006) explore the role of shrouded attributes where a fraction of the consumers are Bayesian, while the remaining consumers are myopic, failing to take into account the cost of shrouded add-ons.

Finally, models can be distinguished on the basis of the nature of the information sent: hard versus soft. Hard information is actually verifiable, while soft information is not. Cheap talk models typically rely on soft information (Crawford and Sobel, 1982) which credibility is increased in the presence of multiple and alternative message dimensions (Chakraborty and Harbaugh, 2014), while models that exploit the strategic use of verifiable information (Milgrom and Roberts, 1986) can consider a verification cost (Caillaud and Tirole, 2007), full and costless verifiability (Glazer and Rubinstein, 2006) or only partial verifiability with the receiver deciding which part to be verified (Glazer and Rubinstein, 2004).

Psychology. There is a large body of psychological evidence suggesting that persuasion activities exploit the fact that individuals have two distinct ways of processing information when they receive a message and have to take decisions based on it (Chaiken and Trope, 1999). Theories in cognitive and social psychology that refer to this idea are typically labelled as dual process theories (Evans, 2003), and the two ways of processing information are also called System 1 and System 2. Kahneman (2003) refers to System 1 and System 2 as, respectively, *intuition* and *reasoning*. Recent neurological research (Goel et al., 2000) suggests that different parts of our brain are actually activated when using System 1 and System 2, respectively. Dual process theories have been applied to explain human behaviors in different setups (Gawronski and Creighton, 2013): persuasion, attitude-behavior relations, prejudice and stereotyping, impression formation, dispositional attribution.

There are two workhorse models of persuasion in psychology. One is the Elaboration Likelihood Model (ELM) (Petty and Cacioppo, 1986a), where the decision-maker can use the "central route" – characterized by a high cognitive effort – or the "peripheral route"– characterized by a low effort. These two routes can be understood as an approximation of a continuum of elaboration intensities which a subject can use when processing information: the higher an individual's cognitive effort, the more likely that she processes all relevant information. At the extremum characterized by highest level of elaboration individuals use all available information and integrate it with already stored information. On the contrary, at the extremum characterized by lowest level of elaboration individuals minimally scrutinize relevant information, extensively using short-cuts to process information.

The other model is the Heuristic-Systematic Model (HSM) (Chaiken et al., 1989), where the decision-maker can use "systematic elaboration" – characterized by careful scrutiny – or "heuristic elaboration" – characterized by the use of simple heuristics, rule of thumbs and categorizations. The basic idea is very similar to that of the ELM. A fully systematic processing of information requires high cognitive effort and considers all relevant information. In contrast, a purely heuristic processing requires minimal cognitive effort and considers only a small amount of information.

Marketing. Persuasion activities are a core subject of marketing research. Often marketing scholars have borrowed models of persuasion from psychology. Indeed, both the ELM and the HSM have been widely applied and tested in the marketing literature (see Shrum et al., 2012, for a recent survey).

An important exception to this is the Persuasion Knowledge Model (PKM), proposed by Friestad and Wright (1994), which has been developed for and dedicated to marketing. In principle, the PKM could be applied to persuasion activities in general, but applications so far have been focusing almost exclusively on marketing (see Campbell and Kirmani, 2008, for a recent review). In the PKM the persuasion process has two actors: the intended target of the persuasion attempt (the consumer) the agent that attempts the persuasion (the marketer). An original feature of the PKM is that it recognizes the strategic behavior of consumers subject to persuasion activities (as documented by, e.g., Kirmani and Wright, 1989; Kirmani, 1990). In particular, consumers are assumed to learn marketers' persuasion tactics over time and, by doing so, consumers become better able to react in their own interest to persuasion attempts (Hardesty et al., 2007). More precisely, the consumer "best replies" to the marketer's persuasion attempt by optimally allocating her (scarce) cognitive resources to the processing of three different dimensions of the persuasive message: information about the persuasion attempt taking place, about the marketer, and about the characteristics of the marketed product. This feature is well captured by the model discussed in the present paper.

Another important feature of the PKM is the recognition of the strategic interaction between marketers' persuasion activities and consumers' reactions. As documented by Kirmani and Campbell (2004), when such interaction is repeated over time it produces a joint dynamics of the behavior of marketers and consumers which can be understood as the result of the contrasting efforts to, respectively, persuade and cope with persuasion attempts (see Wright et al., 2005, for a review on, e.g., the evolution of young consumers' strategies). We cannot explore this aspect within the model presented in this paper, since it is statical by construction. We leave the exploration of dynamics for future research.

Rational allocation of scarce cognitive resources. From a broader perspective, our paper belongs to the small but growing body of literature that tries to model the scarcity of human cognitive resources in a tractable and meaningful way, with the central idea that such a scarcity generates an allocation problem that the decision-maker solves rationally. Even if there are notable exceptions (see, e.g., Shugan, 1980, for an analytical measure of the cost of comparing alternatives), only recently there has been an upsurge of interest in the subject. In particular, a few recent papers have explored the possibility of incorporating the cost of reasoning into models where agents have to take decisions based on the elaboration of information.

One important paper in this regard is Dewatripont and Tirole (2005), that we have already discussed in the Introduction. Another interesting attempt is the model of decision-making proposed by Dickhaut et al. (2009), where the informativeness of a signal (about the payoff associated with different options) increases in the effort that the decision-maker puts in the observation of the signal. Our model is similar in the modeling of cognitive costs, but adds the possibility of the strategic use of the signal by the sender and the explicit modeling of what happens under low cognitive effort – i.e., coarse thinking.

An alternative modelization which is explicitly based on dual process theories of decision-making is provided by Achtziger and Alós-Ferrer (2014) where decisions are the result of two interacting decision processes. One process is controlled, slow and cognitively costly, and it produces a Bayesian updating of beliefs. The second process is automatic, fast and cognitively cheap, and it produces a sort of reinforcement learning that relies on past performance.[6] Our model is similar in spirit, but differs in the focus: while Achtziger and Alós-Ferrer (2014) explore the timing of decisions we study the outcome in a persuasion setup.

Brocas and Carrillo (2012) model how the human brain governs both memorization and retrieval of information from memory by assuming that more precise memorization requires more cognitive resources. Our model obtains similar insights – stronger beliefs on the state of the world lead the agent to exert less cognitive effort to acquire new information – in a choice situation where the relevant signal is not retrieved from memory but chosen strategically by

[6] See also Alos-Ferrer (2015) for an application to self-control and Alos-Ferrer and Ritschel (2015) for further experimental evidence.

a persuader.

In the literature on k-level reasoning, Alaoui and Penta (2013) introduce costs to access higher levels of reasoning, and perform a cost-benefit analysis that allows to endogenize the level of reasoning; pursuing this line, Alaoui and Penta (2014) test this model in the lab and find evidence that supports the idea that players weigh the value of thinking deeper against the cost of reasoning. Our model resembles theirs in that there are explicit cognitive costs but, while in our model the costly activity is about information processing, in their models the costly activity is the iteration of the calculation of best-replies.

Matějka and McKay (2012), following Sims (2003), model the cost of attention by means of constraints on measures of uncertainty – e.g., Shannon entropy.[7] By putting constraints on the amount of cognitive resources that can be devoted to information processing, they introduce elaboration costs that depend on the overall uncertainty reduction, leaving the agent free to allocate such uncertainty on the possible states of the world.[8] On the contrary, we give the agent only the choice between high and low cognitive effort, assuming that when uncertainty is large – i.e., under low elaboration – the agent relies on coarse thinking; this crucially gives to the persuader enough room to exploit reference cues.

Martin (2012) studies the case of a monopolist that independently interacts with rationally inattentive consumers, showing that the monopolist can strategically exploit the consumers' cost of attention. This paper is close to ours in allowing the strategic exploitation by the sender of the receivers' cognitive limitations; however, while we posit that under low elaboration the receivers uses a specific heuristic which is known to the sender – i.e., coarse thinking – in Martin (2012) the receiver can choose how to allocate attention, even when it is very limited.

Caplin and Martin (2014) characterize the testable implications of a model with imperfect perception, providing a unifying framework for all these contributions in the spirit of revealed preference approach. In particular, their framework can easily accommodate dual process reasoners.

Manzini and Mariotti (2014) characterize the testable implications of a model where there is a boundedly rational agent who suffers from limited attention. Although their focus is on

[7]Shannon entropy is typically used in the rational inattention literature as a measure of the uncertainty due to a limited amount of attention (see Wiederholt, 2010, for a review).

[8]A related body of literature considers the case where the decision-maker's attention is drawn to those payoffs or characteritics which are most different or salient relative to the average or to a reference point (Bordalo et al., 2012; Kőszegi and Szeidl, 2013; Bordalo et al., 2013). In (Gennaioli and Shleifer, 2010) only the information relative to the most representative scenarios is retrieved from memory – i.e., a form of local thinking is analyzed.

stochastic choice data, their model is consistent with the idea that the agent has to incur a cognitive cost to better process the available information.

3 The model

We introduce the model in three steps. Firstly, we describe the message received by the decision-maker (to whom we refer as “she”) and how she can elaborate the information it contains. Secondly, we study her behavior with respect to both the choice of the elaboration level and the reaction to the offer. Finally, we introduce the persuader (to whom we refer as “he”) and we analyze his strategic choice regarding the reference cue.

3.1 Message processing: High and low elaboration

The basic working of information processing, and its connection to both the Elaboration Likelihood Model (ELM) and the Heuristic-Systematic Model (HSM), can be summarized as follows. The decision-maker (DM) receives a message – aiming at influencing her attitude towards an offer – which contains information that can be processed at two levels of elaboration. Different elaboration levels are associated with different levels of cognitive effort. If the DM exerts a high cognitive effort, then she can scrutinize the message carefully and extract precise and specific information regarding the offer. Instead, if the DM exerts low cognitive effort, then she processes the message with simple heuristics based on broad categorizations. High cognitive effort can be understood as the use of the “central route” in the ELM or as use of the “systematic elaboration” in the HSM. Low cognitive effort can be understood as the use of the “peripheral route” in the ELM or the use of the “heuristic elaboration” in the HSM.

More formally, message processing is modeled as follows. The DM faces a two-part message (q, r) associated with an offer which she has to decide upon. Part $r \in R$, of the message is a reference cue, i.e., a piece of information which allows the DM to refer the offer to a specific category of offers and, through this, to infer the expected quality of the offer she is facing. We denote with R the set of all possible reference cues and, therefore, categories. Part $q \in \{G, B\}$ of the message contains the information regarding the actual quality of the offer: if $q = G$ then quality is good, while if $q = B$ then quality is bad.

Whenever the DM is aware of a message (q, r), she initially processes it under low elaboration, i.e., $e = L$, so observing immediately the reference cue r, while the actual quality q of the offer remains unknown. Such elaboration is *automatic*, in the sense that it cannot be avoided by the DM. At this stage the DM has a belief about the quality of the offer that depends on her prior about quality as well as on the cue r. Given such a belief, the DM evaluates whether to increase effort to $e = H$for high elaboration, paying c_e and acquiring the knowledge of q. The use of high elaboration is hence *deliberate* and *controlled*, possibly triggered by the observation of the cue, but also potentially able to drastically change DM's beliefs with respect to those induced by the cue.

We emphasize that part r of the message is informative in the sense that it refers to a category of objects of which DM knows the average quality. This is a simple way to model the fact that, when choosing $e = L$, DM is affected by "coarse thinking", i.e., DM puts different offers with a common characteristic in the same mental category and treats them all in the same way (Fryer and Jackson, 2008; Mullainathan et al., 2008).

Figure 1 is a graphical representation of the different information on quality that can be drawn from the same message depending on the elaboration level chosen by DM.

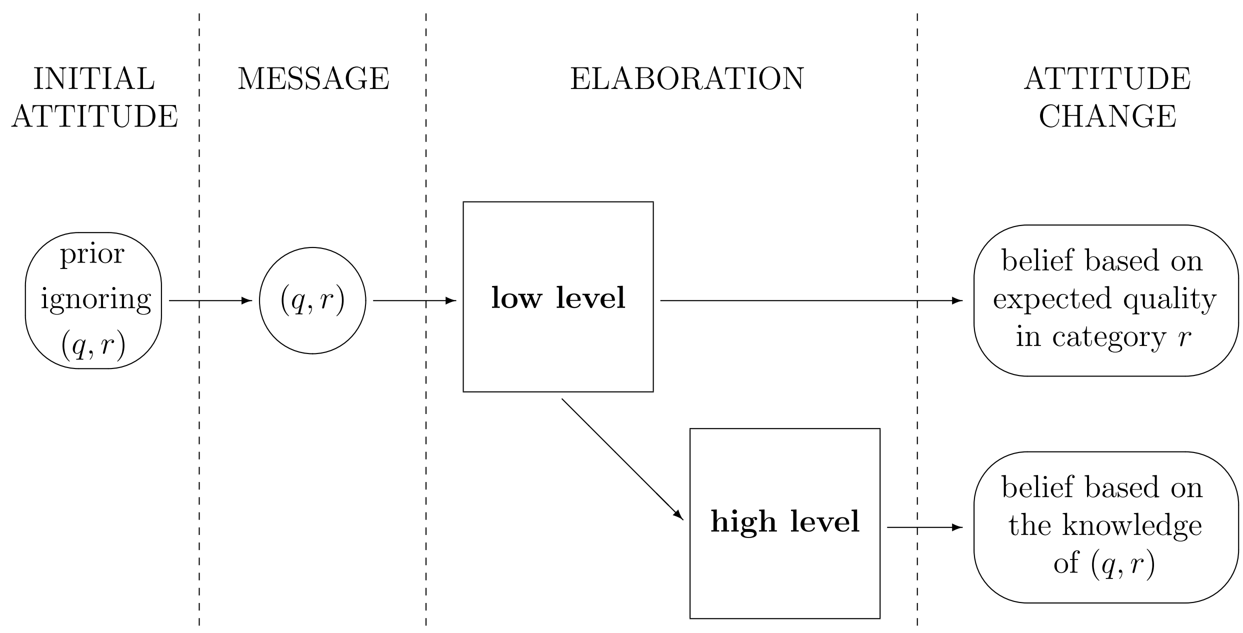


Figure 1: A graphical representation of message processing. Two elaboration levels are possible: high elaboration H which allows to observe (q, r) and low elaboration L which allows to observe only r.

3.2 Decision-maker behavior: Optimal elaboration and decision

After having processed the message, the DM has to decide whether to accept the offer, a case denoted with Y, or reject it, denoted with N.

The utility obtained by the DM depends on the actual quality of the offer and the exerted level of cognitive effort. If the DM accepts the offer when $q = G$, then she obtains $\gamma U_G > 0$. If instead she accepts the offer when $q = B$, then she obtains $\gamma U_B < 0$. Here, $\gamma > 0$ is a parameter that represents the motivation of the DM, i.e., her perception of how much stake is involved in the offer. If the DM rejects the offer, then she obtains a null utility independently of q. In any case, if the DM exerts $e = H$, then she also has to bear the elaboration cost c_e.[9]

From the structure of DM's utility immediately follows that, if DM exerts $e = H$, then she finds it profitable to choose Y in case $q = G$ and N in case $q = B$. We indicate such behavior with HYN. We denote behaviors that make use of low elaboration with LY and LN, respectively leading to acceptance and rejection.

We now proceed to compare the expected utility that DM obtains by choosing HYN, LY, and LN. We denote with $\mu_r \in [0, 1]$ the posterior belief – after the observation of cue r – that the offer is of quality G. Given a belief μ_r, we obtain that the choice of LY leads to an expected utility of $\mu_r \gamma U_G - (1 - \mu_r)\gamma |U_B|$, the choice of HYN leads to an expected utility of $\mu_r \gamma U_G - c_e$, and the choice of LN leads to an expected utility of 0. By solving, with respect to μ_r, for the highest expected utility across the previous ones, one gets that DM's expected utility is maximized by HYN when $\frac{c_e}{\gamma U_G} \leq \mu_r \leq 1 - \frac{c_e}{\gamma |U_B|}$, by LY when $\mu_r \geq \max\left\{1-\frac{c_e}{\gamma |U_B|}, \frac{|U_B|}{U_G+|U_B|}\right\}$, and by LN when $\mu_r \leq \min\left\{\frac{c_e}{\gamma U_G}, \frac{|U_B|}{U_G+|U_B|}\right\}$.[10] The optimal behavior by DM as a function of p is summarized in Figure 2. This leads to the following result (of which we omit the trivial proof):

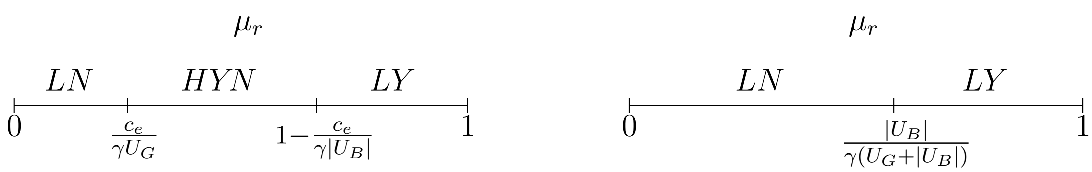
PROPOSITION **1** (Decision-maker's optimal behavior)**.**
Let μ_r be the decision-maker's belief about the offer quality q after her automatic low elab-

[9]In this paper we stick to the interpretation that Y means acceptance and N means rejection. However we stress that other interpretations are possible. For instance, if the offer is a consumer good to be bought, then Y could mean to buy a large quantity of the good and N to buy a small quantity; also, Y could mean to pay a high price per unit while N to pay a low price per unit. Both cases can easily be accommodated by the model since what really matters for the results is that Y is preferred when quality is G and N is preferred when quality is B. We observe that U_G and U_B can be interpreted as the relative gains of choosing Y over N when, respectively, quality is G and quality is B.

[10]Note that HYN turns out to be optimal for some intermediate range of values of μ_r only if $c_e \leq \frac{U_G |U_B|}{U_G+|U_B|}$, although when equality holds there is just one value of p for which HYN is optimal and for that value DM is indifferent between HYN, LN, and LY.

oration of the persuader's message (q, r)*, leading to the observation of the reference cue* r*. Then, the optimal behavior of the decision-maker is:*

- *to engage in further high elaboration of* (q, r) *to acquire knowledge of* q*, and subsequently accept the offer if* $q = G$ *and reject it if* $q = B$*, when* μ_r *is not too extreme and* c_e *is sufficiently low;*
- *to stay with low elaboration of* (q, r) *and accept the offer when* μ_r *is sufficiently high;*
- *to stay with low elaboration of* (q, r) *and reject the offer when* μ_r *is sufficiently low.*



(a) when $c_e < \frac{U_G|U_B|}{U_G+|U_B|}$.

(b) when $c_e > \frac{U_G|U_B|}{\gamma(U_G+|U_B|)}$.

Figure 2: DM's optimal behavior as a function of μ.

Intuitively, the DM is less likely to choose H if the elaboration cost c_e is higher, and never uses it if c_e is too high. Moreover, the expected benefits for the DM to choose H over L decrease when beliefs on quality are more extreme – i.e., a high μ_r that gets closer to 1 or a low μ_r that gets closer to 0 – because less and less uncertainty remains while the same elaboration cost has to be borne. So, the more extreme the beliefs induced by the reference cue, the more likely that the DM does not engage in high elaboration and takes action on relying on the information provided by the cue.

3.3 Persuader behavior: Strategic use of reference cues

We now introduce the persuader (P) and focus on the strategic use of the reference cue r. In particular, we study how P can induce in the DM favorable beliefs about his offer through r. This behavior on the part of P can be interpreted as an attempt to "frame" the offer (see, e.g., Mullainathan et al., 2008): the observation of r induces the DM to associate P's offer to a specific mental category containing all offers sharing characteristic r, and consequently to evaluate P's offer by considering the average quality of offers in such a category.

Formally, there is an initial stage of the game where the source of the offer is determined. With probability $\alpha_x > 0$ the offer is not made by P and belongs to the category associated with reference cue x, while with probability $\alpha_y > 0$ the offer is not made by P and belongs to the category associated with reference cue y. Further, with probability $\alpha_P = 1 - \alpha_x - \alpha_y > 0$ the offer is made by P. Probabilities α_x, α_y, and α_P are intimately related to coarse thinking: they can be interpreted as relative frequencies of occurrence and, hence, they are naturally thought of as dependent on the relative size of categories associated with x and y, which in turn is affected by the degree of coarse thinking (more details on this in Appendix C.1).

If the offer does not come from P and belongs to category x, then it is of quality $q = G$ with probability β_x. Similarly, if the offer does not come from P and belongs to category y, then it is of quality $q = G$ with probability β_y. Hence, the parameters β_x and β_y represent the fraction of good quality offers in, respectively, category x and y, not taking into account the behavior of P. Without loss of generality we assume that $\beta_x > \beta_y$, i.e., on average x refers to a higher quality than y.[11]

Instead, if the offer comes from P, then the game unfolds as follows: the quality of the offer – i.e., P's type – is $q = G$ with probability α_G and $q = B$ with probability $\alpha_B = 1 - \alpha_G$, and then P chooses a reference cue $r \in \{x, y\}$ to be associated with the offer. Probabilities α_G and α_B can be interpreted as, respectively, the fraction of good quality offers and the fraction of bad quality offers when P is called into play. We remark that the quality of an offer should be interpreted in a broad sense, as something that correlates positively with DM's utility. The cost for P of choosing reference cue r is c_r. Since $\beta_x > \beta_y$, to rule out uninteresting cases we assume that $c_x > c_y = 0$, i.e., we posit that referring the offer to a category of higher average quality is more costly for P, and we normalize c_y to zero. Quality q is known to P, so a strategy for P is a function $\rho : \{G, B\} \longrightarrow \{x, y\}$ indicating which reference cue is chosen conditionally on the quality of the offer. The utility for P is $V > c_x$ in case DM accepts the offer while it is 0 if DM rejects the offer. In any case, the cost c_r must be borne. We stress that the zero utility obtained by P when DM chooses N should be seen as a normalization, so to be consistent with different interpretations of Y and N; for instance, if N means to buy at a low price or a small quantity, then P would earn a low but possibly positive profit.

The choices that maximize P's utility are easily established, since P obtains V only if DM reacts with Y, while P incurs the cost of reference c_r independently of DM's choices. So, P wants the offer to be accepted by DM, but while when $q = G$ it is enough that DM plays HYN, when $q = B$ it is necessary that DM plays LY. Also, everything else being equal, P strictly prefers to send cue y, since it costs less than cue x. This leads to the following proposition summarizing P's best replies to the optimal choices by the DM, as described in

[11] An example showing how to endogenize β_x and β_y is provided in Appendix D.3.

Proposition 1 (the proof can be found in Appendix A):

PROPOSITION **2** (Persuader's optimal behavior)**.**
The optimal behavior of the persuader is:

- *to send cue y, when the decision-maker reacts in the same way to cue x and cue y;*
- *to send cue x, when the decision-maker reacts to cue x by staying with low elaboration and accepting the offer, while she reacts to cue y by staying with low elaboration and rejecting the offer;*
- *to send cue x if $q = G$ and cue y if $q = B$, when the decision-maker reacts to cue x by engaging in high elaboration, while she reacts to cue y by staying with low elaboration and rejecting the offer;*
- *to send cue x if $q = B$ and cue y if $q = G$, when the decision-maker reacts to cue x by staying with low elaboration accepting the offer, while she reacts to cue y by engaging in high elaboration.*

Importantly, since we now explicitly consider the choice of reference cue r made by P, it follows that the actual average quality of offers in categories X and Y is also endogenous, partly depending on P's choice. Let us introduce some further notation to handle this fact in the subsequent analysis. We denote with $\hat{\beta}_r$ the average quality of offers in the category associated with the reference cue r when P's choices are taken into account.[12]

4 Persuasion equilibria

Section 3 provides a description of the optimal behavior for both the persuader and the decision-maker. However, in order to make predictions in line with rationality and Bayesian beliefs, we need to identify the *equilibrium* behaviors, which in this case requires looking for joint behavior optimality under beliefs that are updated according to Bayes' rule (whenever possible) – that is, we have to consider the Bayes-Nash equilibria of the described model.

Preliminarily, we note that as long as α_x, α_y, β_x, β_y are strictly comprised between 0 and 1, the DM faces each combination of offer quality and reference cue with positive probability. Therefore, every Bayes-Nash equilibrium – to which we simply refer as *persuasion equilibrium* or simply *equilibrium* – is also sequential, and, hence, weak perfect Bayesian.[13]

[12]The values taken by $\hat{\beta}_r$ depend on P's choices as described by function ρ. The different possible values of $\hat{\beta}_r$ are summarized in Appendix A.

[13]This is because there are no out-of-equilibrium information sets, and hence no out-of-equilibrium beliefs. We also note that such an absence makes it impossible to refine the set of equilibria by applying criteria that rule out some scarcely plausible out-of-equilibrium beliefs (such as, e.g., the Intuitive Criterion or D1).

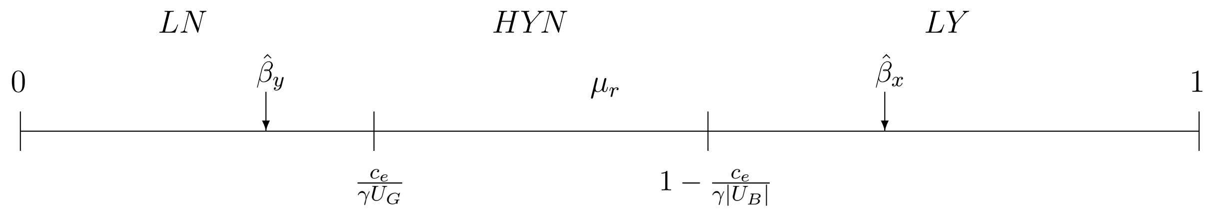


Figure 3: A pooling equilibrium where both G and B send cue x, and DM finds it optimal to reply with LY to cue x and with LN to cue y. Elaborations costs are low enough (i.e., $c_e < \frac{U_G|U_B|}{U_G+|U_B|}$) to make HYN a best reply for intermediate levels of expected quality.

As the first step of the analysis we establish under what conditions an equilibrium exists and is unique. Indeed, we might have that, at least for some parameter values, no equilibrium exists, or that more than one equilibrium is possible. It turns out that the *strength* of coarse thinking – i.e., the *coarseness* of the categorization made by the decision-maker – plays an important role in guaranteeing both existence and uniqueness of persuasion equilibria. This is summarized by the following proposition, which proof can be found in Appendix A:

Proposition **3** (Existence and uniqueness of persuasion equilibrium)**.**
If coarse thinking is strong enough, then almost always an equilibrium exists and is unique.

In order to understand why a stronger coarse thinking can guarantee the existence and uniqueness of a persuasion equilibrium, it is important to note that inexistence and multiplicity share a common root: the endogeneity of expected quality $\hat{\beta}_x$ and $\hat{\beta}_y$, and in particular their dependence on the behavior of P. Indeed, if $\hat{\beta}_x$ and $\hat{\beta}_y$ are very sensitive to P's choices, then it can happen that none of the possible pairs of posteriors $\mu_x = \hat{\beta}_x$ and $\mu_y = \hat{\beta}_y$ is compatible with equilibrium, or that more than one pair is. Importantly, the degree of beliefs' endogeneity is mitigated by coarse thinking, namely by the fact that under low elaboration DM cannot distinguish an offer that comes from P from an offer of the same category that does not come from P. In particular, the dependency of DM's posteriors on P's behavior decreases as coarse thinking gets stronger because the offer categories X and Y widen, and hence the individual impact of P's behavior becomes less relevant for the average quality of each of them. In the limit where categories X and Y becomes infinitely large, $\hat{\beta}_x$ and $\hat{\beta}_y$ converge to β_x and β_y, respectively, whatever P's choices are.

From now on we assume that coarse thinking is strong enough to guarantee that a unique persuasion equilibrium exists, and we turn our attention to the characteristics that such an equilibrium can have. As it is typical of communication models, there two main kinds of equilibria: *pooling* equilibria where P always send the same cue – independently of the offer quality – and a *separating* equilibria where P sends two distinct cues – one when the offer is of good quality and another one when it is of bad quality. Moreover, in the present model pooling and separating equilibria can potentially be of two different kinds each. We can have pooling equilibria where P always sends x or always sends y. Also, we can have separating equilibria where P sends x when the offer is of good quality and y when the offer is of bad quality, or viceversa.

It turns out that all four kinds of equilibria can occur, depending on the model parameters. We do not give here the exact conditions on parameters the make a particular kind of equilibrium possible (we refer the interest reader to Appendix B). Instead, we give an example for each kind of equilibrium, providing a graphical illustration in terms of R's beliefs and optimal behavior.

Figure 3 illustrates a pooling equilibrium where P always sends the costly cue x. The cost of elaboration is low and the belief associated with cue x is not too high, so that the DM chooses HYN when she sees cue x, while the belief associated with y is quite low, so that the DM choose LN when she sees cue y.

A pooling equilibrium where P sends always the cheap cue y is depicted in Figure 4. Elaboration costs are low and the beliefs associated with both x and y are not very extreme, so that DM reacts by choosing HYN upon seeing either x and y. Since DM chooses her action independently of cue, P finds it optimal to save on cue costs, always sending y.

Figure 5 illustrates an example of a separating equilibrium where P sends the costly cue x when the offer is of good quality while he sends cheap cue y when the offer is of bad quality. Elaboration costs are low enough and beliefs associated with x not too high as to induce DM to choose HYN when she sees cue x; at the same time, the belief associated with cue y are so low as to induce DM to choose LN. So, the persuader finds it optimal to incur the cost of sending x when the offer is of good quality, since this leads his offer to be accepted, while he prefers to save on costs and send the cheap cue y when the offer is of bad quality,

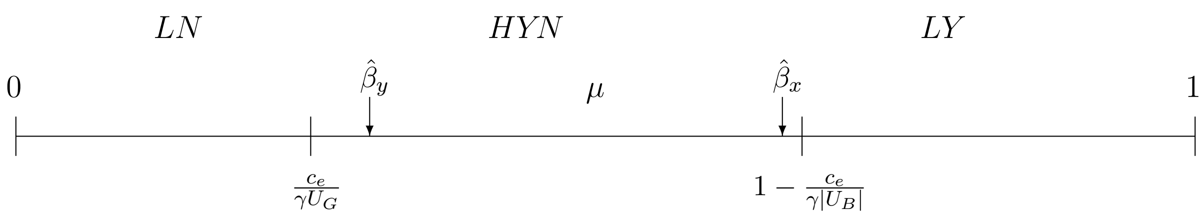


Figure 4: A pooling equilibrium where both G and B send cue y, and DM finds it optimal to reply with HYN to both cue x and y. Elaborations costs are low enough (i.e., $c_e < \frac{U_G|U_B|}{U_G+|U_B|}$) to make HYN a best reply for intermediate levels of expected quality.

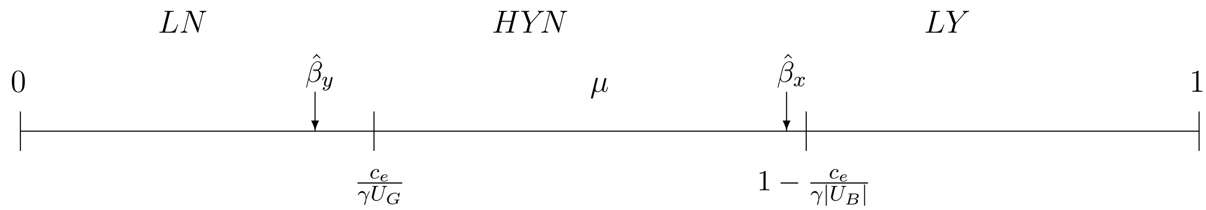


Figure 5: A separating equilibrium where G sends cue x and B sends cue y, and DM finds it optimal to reply with HYN to cue x and with LN to cue y. Elaboration costs are low enough (i.e., $c_e < \frac{U_G|U_B|}{U_G+|U_B|}$) to make HYN a best reply for intermediate levels of expected quality.

since in no case his offer will be accepted.

The alternative kind of separating equilibrium is depicted in Figure 6. Also in this case elaboration costs are low, but the belief associated with x is high enough as to induce DM to choose LY when she sees cue x; moreover, the belief associated with cue y are not too low, as to induce DM to choose HYN. Therefore, the persuader finds it optimal to incur the cost of sending x when the offer is of bad quality, since this leads his offer to be accepted without much scrutiny, while he prefers to save on costs and send the cheap cue y when the offer is of good quality, since in this case his offer will be accepted after careful scrutiny.

5 Behavioral implications and consistency with psychological findings

The way in which we have modeled reference cues and costly elaboration turns out to be consistent with a number of findings of dual process models of persuasion – and, in particular, of the ELM and the HSM. For clarity we organize them in remarks.

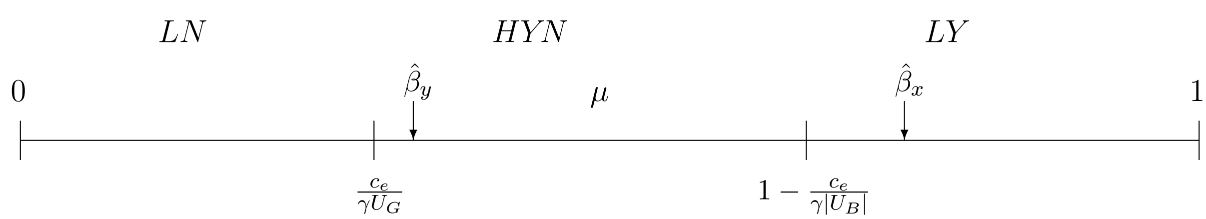


Figure 6: A separating equilibrium where G sends cue y and B sends cue x, and DM finds it optimal to reply with LY to cue x and with HYN to cue y. Elaboration costs are low enough (i.e., $c_e < \frac{U_G|U_B|}{U_G+|U_B|}$) to make HYN a best reply for intermediate levels of expected quality.

Remark **1.** *The decision-maker interprets the persuasive message differently under high elaboration and under low elaboration (Petty and Cacioppo, 1986a; Eagly and Chaiken, 1993).*

This fundamental idea of dual process theories of information elaboration is directly embedded in the way DM can extract information from the message (q, r): under low elaboration only r is observed, while under high elaboration also q is. We stress that the interpretation of the message is different even if low and high elaboration lead to the same choice regarding the offer. Indeed, the DM holds different beliefs when under L and under H, with those obtained under H being typically much more extreme — thanks to the extra information. As discussed below, this has important consequences on the stability of attitudes when further messages are received.

Remark **2.** *The recourse to high elaboration is more likely if the DM's motivation (or need for cognition) is higher, even if motivation does not affect the relative desirability of good versus bad offers (Petty and Cacioppo, 1979, 1981; Cacioppo et al., 1983; Petty et al., 1981).*

To see why it is so, consider an increase in the parameter γ, fon instance from γ' to $\gamma'' > \gamma'$. Such increase in motivation has two distinct effects, which both make the recourse to high elaboration more likely. The first effect is that H becomes optimal for a wider range of values of U_G and U_B, making c_e more likely to induce a switch from the case depicted in panel (b) of Figure 2 to the case depicted in panel (a). This follows from the fact that $\gamma'' > \gamma'$ implies that $\gamma' U_G |U_B|/(U_G + |U_B|) < \gamma'' U_G |U_B|/(U_G + |U_B|)$. The second effect is that H becomes optimal for a wider range of beliefs on quality, including now beliefs that are a bit more extreme – as depicted in Figure 7. This follows from the fact that $\gamma'' > \gamma'$ implies that $c_e/(\gamma' U_G) > c_e/(\gamma'' U_G)$ and $1 - c_e/(\gamma' |U_B|) < 1 - c_e/(\gamma' |U_B|)$. It must be noted that there are no effects on the likelihood of acceptance of the offer under L, since the threshold $|U_B|/(U_G + |U_B|)$ is unaffected by γ. Hence, motivation modifies the likelihood of acceptance only through its impact on the likelihood of high elaboration.

Remark **3.** *A greater ability to think and focus on the content of the message makes it more likely that the decision-maker recurs to high elaboration (Petty et al., 1976; Petty and Cacioppo, 1986b).*

Better cognitive skills of the DM are represented in our model by a smaller elaboration cost c_e associated with the use of H. It must be noted that, if we restrict attention to a single decision by the DM, the effects in terms of attitude change induced by a decrease in c_e are qualitatively the same as those induced by an increase in γ, since a reduction in c_e shifts the

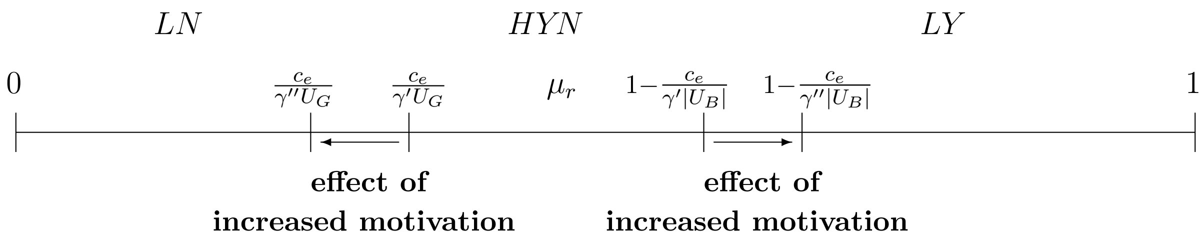


Figure 7: Effect of an increase in DM's motivation (or need for cognition) from γ' to $\gamma'' > \gamma'$.

relevant thresholds in the same direction of an increase in γ. Nevertheless, our model does not predict an observational equivalence between a reduction in c_e and an increase in γ: a given individual has a fixed c_e for a fixed complexity of the message, while she typically has a value of γ that depends on the nature and context of the offer. So, it remains possible to separate the effect of c_e from the effect of γ by having the same DM receiving messages of fixed complexty but concering offers towards which DM is heterogeneously motivated.

Remark **4.** *The persuader can use cues to affect the decision-maker's choice of the elaboration level and, through it, the decision-maker's attitudes (Petty and Cacioppo, 1986b; Chaiken and Trope, 1999).*

To see how P can change the attitudes of the DM by means of reference cues consider the case where $c_e < \gamma U_G |U_B|/(U_G + |U_B|)$ and the DM's posterior associated with y is $\mu_y \in [c_e/(\gamma U_G), 1 - c_e/(\gamma |U_B|)]$. So, without additional information beyond the observation of cue y, DM's attitude would be to *accept*, if μ_y is larger than $|U_B|/(U_G + |U_B|)$ and to reject if it is *smaller*. Moreover, the DM would like to gather additional information by *engaging in high elaboration*, and to condition acceptance on the outcome of the elaboration. Suppose now that upon observation of cue x the DM forms the posterior belief $\mu_x \in (1 - c_e/(\gamma |U_B|), 1]$. So, when the DM sees x, she decides to *accept the offer without engaing in high elaboration*. Hence, if $\mu_y < |U_B|/(U_G + |U_B|)$ and $q = B$, P can send x instead of cue y and so doing he is successful in changing the attitude of DM from *reject* to *accept* and in *preventing high elaboration*. This leads to separating equilibrium where P send cue y only when his offer is of bad quality.

Remark **5.** *Persuasion under high elaboration is stabler than persuasion under low elaboration (Petty and Cacioppo, 1986b; Haugtvedt and Petty, 1992).*

Attitude change is less stable under low elaboration because the posterior obtained without engaging in high elaboration are more sensistive to new information regarding the offer. To see this consider the case where the DM receives additional information about the offer in the form of a private signal that with probability σ reveals q. The DM observes the signal subsequently to the choice of the elaboration level, but prior to the choice to accept or reject the offer. As a consequence, LY leads to accept only if no signal is received or the signal reveals $q = G$, LN leads to reject the offer only if no signal is received or the signal reveals $q = B$, while HYN leads exactly to the same behaviors as without the possibility of additional information – that is, behavior is independent of the signal. In particular, with respect to the case without additional information we have that: under HYN, the DM maintains the same decision to accept or reject with probability 1; under LY, the DM maintains the same decision to accept with probability $1-\alpha_s+\alpha_s\beta < 1$; under LN, the DM maintains the same decision to reject with probability $1-\alpha_s+\alpha_s(1-\beta) < 1$.[14]

Remark **6.** *Prejudice can affect the elaboration level and induce biased elaboration (Petty and Cacioppo, 1986b; Petty et al., 1999).*

Prejudice can be formalized by allowing DM to have biased priors about the relative size of offer categories, i.e., α_x, α_y, the average quality in each category, i.e., β_x and β_y, and the likelihood that P has a good quality offer, i.e., α_G. Any such bias leads to the calculation of expected qualities $\tilde{\beta}_x$ and $\tilde{\beta}_y$ which are biased with respect to the correct ones, i.e., $\hat{\beta}_x$ and $\hat{\beta}_y$. The DM's decision to engage in high elaboration is hence affected by prejudice, as it is based on $\tilde{\beta}_x$ and $\tilde{\beta}_y$. For instance, if DM has a prejudice against x, i.e., $\tilde{\beta}_x < \hat{\beta}_x$, then she could choose HYN in place of LY or LN in place of HYN. Further, prejudice can have effects even after high elaboration of the message whenever H does not allow to observe q with certainty, but instead it allows to learn q with probability between 1/2 and 1. This case, which represents the more reasonable situation where effortful elaboration does not guarantee perfect knowledge of the offer quality, is explored in greater detail in the extension of the model presented in Section 7 where the intensity of elaboration is assumed to be vary continuously in the elaboration effort.

Remark **7.** *The persuader can send arousing and other mood-affecting cues to induce low elaboration (Petty and Cacioppo, 1986b; Sanbonmatsu and Kardes, 1988).*

A reference cue can have the purpose to induce arousal (or other moods) in the DM, with the aim of increasing the likelihood that the DM relies on low elaboration. This is represented in our model as a cue which, upon observation, increases the elaboration cost c_e associated with the use of H. If the persuader has can send an arousing cue (or other mood-affecting factors),

[14] We also note that, incidentally, α_s acts against HYN, similarly to an increase of c_e.

he can use it with the aim to increasing the likelihood that DM relies on low elaboration. To formalize this idea let us add, besides x and y, a further characteristic of the offer: a mood cue that we indicate with $m \in \{a, n\}$, where $m = a$ means that the cue induces arousal and $m = n$ that it does not. In this setup there are four offer categories identified by the pairs (x, a), (y, a), (x, n), and (y, n). So, P has to choose not only the reference cue between x and y, but also the mood cue between a and n, where choosing a costs $c_a > 0$ and n costs nothing. Under L, DM observes both r and m, and if $m = a$ the cost of engaging in high elaboration increases from c_e to $c_e^a > c_e$. As a result the persuader can use the mood cue together with a reference cue in oder to make DM stay with low elaboration and, thanks to this, possibly obtain the acceptance of his offer. Figure 8 illustrates a case where the mood cue does not convey information on average quality – i.e., categories (x, a) and (x, n) have the same average quality, and similarly for categories (y, a) and (y, n) – but nevertheless the persuader whose offer is of bad quality would like to send $m = a$. Indeed, P has his offer rejected if he sends (y, n) because DM engages in high elaboration and discovers that the offer is actually of bad quality, while if he sends (x, a) then – thanks to arousal – the DM stays with low elaboration and P gets the offer accepted.

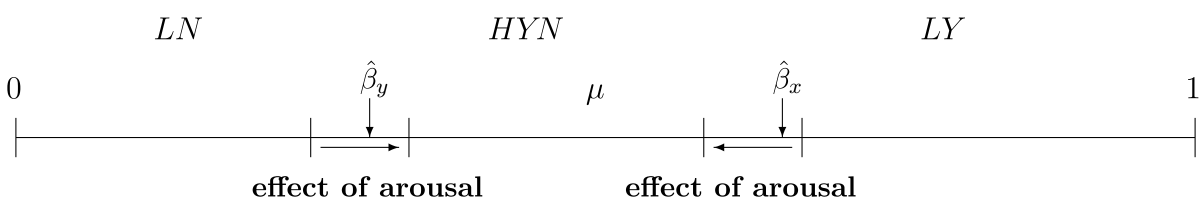


Figure 8: If P sends mood cue n, then the DM finds it optimal to reply with HYN to both reference cues x and y. However, if P sends mood cue a than the range of beliefs for which HYN is an optimal choice shrinks. As a result, if P sends (x, a) then the DM reacts with LY, while if P sends (y, a) then the DM reacts with LN.

6 Applications to economic, social, and political contexts

The model developed in this paper is aimed at improving our undestanding of the phenomenon of persuasion. As such, and despite its simplicity, the model must prove to be flexible enough to fit the variety of persuading activities that, as we have argued in the

choice/market	P	DM	cue x	cue y
food selling	milk producer	consumer	glass bottle	plastic bottle
shop location	retailer	consumer	expensive street	cheap street
financing	entrepreneur	loan agent	expensive dress	cheap dress
essay evaluation	student	teacher	long essay	short essay
job market	candidate	recruiter	many certifications	few certifications
project approval	policy-maker	assembly	merit argument	standard argument
fundraising	fundraiser	donor	face-to-face	by post
social responsibility	corporation	consumer	responsible actions	just stick to law

Table 1: Examples of situations that the model can reasonably fit.

Introduction, are many and widespread in our economic and social life.[15] In the attempt to convince the reader that this is actually the case, we briefly discuss a wide range of situations that reasonably fit the model. Table 1 summarizes them.

As sketched in the Introduction, milk is usually sold to consumers either in glass bottles or in plastic bottles. Glass is more expensive than plastic, but can be used by the seller to induce a higher expectation. Indeed, if consumers exert low elaboration effort and observe a glass bottle containing milk, then they generically think of products in glass containers, whose average quality can be higher than that of products in plastic containers. If consumers choose to rely on high elaboration, then they have to read and understand data on labels, recover information from memory, and put effort to fully assess the quality of the product.

A seller has to decide where to set up his shop, either in the costly location where most shops sell high quality products or in the cheap location where most shops sell low quality products. Consumers know the average quality of the products sold in each location, but in order to assess the quality of any single product they have to scrutinize it carefully, bearing an elaboration cost.

A loan agent working for a financial institution has to decide whether to lend money to

[15]The model can be seen as a natural variant of Crawford and Sobel (1982) where talk is not cheap but costly in a twofold sense: the sender has to sustain a cost to send one signal, x, instead of another one, y, and the receiver has to incur a cost in order to learn what action is best for her, Y or N, without relying on the signal. Obviously, the cost that the receiver has to bear can have a non-psychological nature. It might be a search cost or just a time cost, or even a direct monetary cost to get verifiable information. Although the model fits well these cases too, we stress that we want to focus on situations where the cost that the receiver has to bear is psychological. We leave this alternative intepretation, and the related adjustments to the model and its extesions, for future research.

an entrepreneur. The agent may put effort in studying the details of the investment plan for which the entrepreneur is asking money, but this can turn out to be a task requiring a substantial cognitive effort. Alternatively, the agent can look at how the entrepreneur presents himself considering, for instance, his dressing or his club memberships, and then can base her decision on the average quality of investment plans by similar entrepreneurs. Obviously, an elegant dress is more expensive than a casual one and a more exclusive club has higher fees than an unfashionable one.

A teacher, when marking a student's essay, may spend cognitive resources to scrutinize it carefully, or she can simply choose a mark on the basis of the length of the essay, considering that longer essays are usually better than shorter ones. Writing a longer essay is evidently more time demanding for a student.

A candidate who has applied for a job needs to convince the recruiter that she is qualified for that job. Before the interview the candidate can obtain a bunch of certified qualifications, which require time and effort to get. The recruiter can exert high effort and analyze each qualification in detail, trying to understand what is the real productivity of the candidate. Alternatively, the recruiter can rely on low elaboration and just consider that many qualifications typically go with a good candidate, while a bad candidate has only few qualifications.

A policy-maker who wants to carry out a project which is subject to approval by an assembly has to convince the members of the assembly that the benefits of the project are higher than its costs. The policy-maker can modify the project in such a way that it appears to belong to a category of projects that the assembly tends to consider favourably, e.g., improving environmental protection, defending human rights, or enforcing equality of opportunities. By doing this, however, the policy-maker can incur in a cost due to a constrast with his own preferences or due to opportunity costs in the use of the resources available for the project. The voters in the assembly can exert low elaboration effort and evaluate the project on the basis of their preferences for the relevant category of projects, or otherwise they can exert high effort and analyze the details about costs and benefits.

A fundraiser working for a non-profit organization aims at collecting voluntary contributions from a potential donor to finance a charitable initiative. The fundraiser has a leaflet, full of detailed information about the destination of donations and the trustworthiness of the non-profit organization. He can decide to send the leaflet through the postal service for a very low fare, or to deliver it in person, which is more costly in terms of both time and money. The leaflet contains all the information needed by the donour to assess the merit of the initiative, but the scarce familiarity of the donor with the specific initiative makes it cognitively costly to extract all relevant information. Alternatively, the donor can take a decision on the basis of whether the leaflet was delivered in person or via mail. The average quality of initiatives for which fundraisers have personally talked to the potential donor may well be higher than the quality of initiatives where nobody has shown up.[16]

A corporation produces goods for which consumers are willing to pay a premium price only if such goods are believed to be produced by a truly socially responsible corporation. The actual business model of the firm can be socially responsible or not; consumers have the possibility to investigate this by incurring a cost to both collect and elaborate all relevant information. The corporation chooses whether to make a publicly observable investment that is typically made by socially responsible corporations, or simply stick to law and save on the cost of the investment.

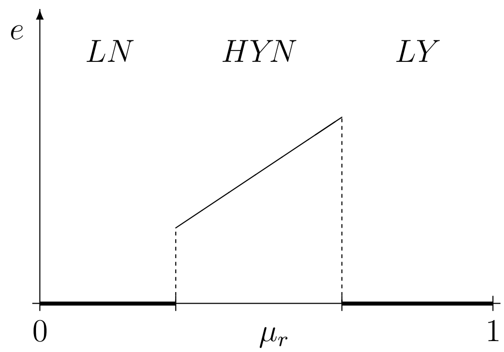
7 A continuum of elaboration intensities

Petty and Cacioppo (1986b) have suggested that the DM might not be constrained to choose sharply between high and low elaboration, but can typically select an elaboration intensity out of an elaboration continuum, where low and high elaboration are the two extremes. The model presented in Section 3 can be easily adapted to this idea.

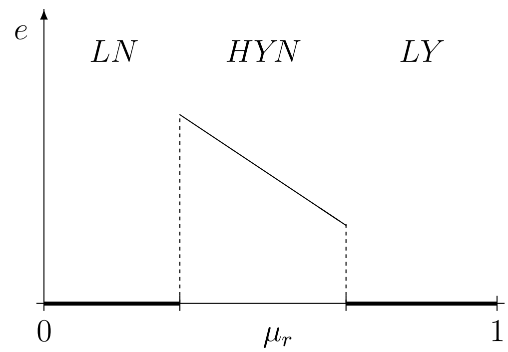
Basically, the amount of information on q extracted from the message should increase continuously in the elaboration effort e that in turn should be a continuous variable. To model this let us assume that, by engaging in high elaboration, the DM does not directly observe q but extracts from the message a signal σ which conveys information on q. Also, the precision of σ depends on the elaboration intensity $e \geq 0$ that the DM chooses. Let $p(e)$ be the probability that the signal reports q correctly, i.e., it says G when the quality is G and B when the quality is B, while $1 - p(e)$ is the probability that the signal is wrong, i.e., it says B when the quality is G and G when the quality is B. Further, let p be differentiable and such that $p'(e) > 0$, which models the fact that the extraction of correct information about q is more likely under greater elaboration intensity, and $p''(e) < 0$, which models the fact that elaboration has increasing marginal costs of effort.

Clearly, choosing a positive level of elaboration makes sense only if the acquired information is then used, i.e., only if the DM chooses HYN. The expected utility of choosing HYN with elaboration intensity e^* is $\mu_r p(e^*)\gamma U_G - (1-\mu_r)(1-p(e^*))\gamma|U_B| - e^*$, while the expected utilities of choosing LY and LN with null elaboration intensity are as in the model of Section 3, i.e., $\mu_r\gamma U_G - (1-\mu_r)\gamma|U_B|$ and 0, respectively. Solving the inequal-

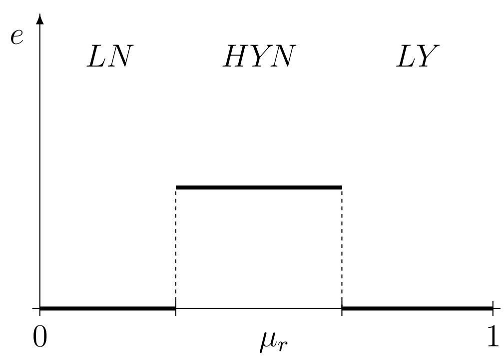
[16] Della Vigna et al. (2012) found in a field experiment that a door-to-door fundraising activity is more effective for charitable purposes.



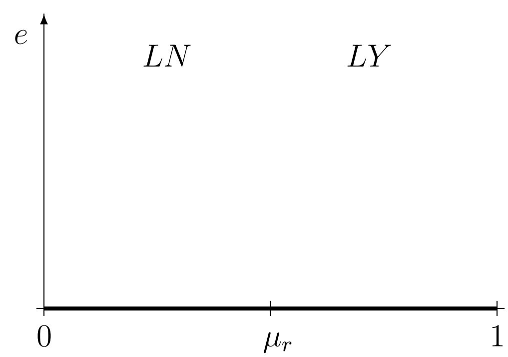
(a) HYN optimal for some μ and $U_G > |U_B|$.



(b) HYN optimal for some μ_r and $U_G < |U_B|$.



(c) HYN optimal for some μ_r and $U_G = |U_B|$.



(d) HYN never optimal.

Figure 9: In subfigures (a), (b) and (c) there exists an interval of beliefs against which HYN is best reply, which is the case for $p(e^*) > 1/2 + \frac{e^*(U_G+|U_B|)}{2\gamma U_G|U_B|}$. Moreover, LN is best reply for beliefs lower than $\frac{e^*+(1-p(e))|U_B|\gamma}{p(e)\gamma U_G+(1-p(e))\gamma|U_B|}$, while LY is best reply for beliefs higher than $\frac{p(e)\gamma|U_B|-e^*}{(1-p(e))\gamma U_G+p(e)\gamma|U_B|}$, and HYN is best reply for intermediate beliefs. In subfigure (d), instead, $p(e^*) < 1/2 + \frac{e^*(U_G+|U_B|)}{2\gamma U_G|U_B|}$ and HYN is never best reply, while LN and LY are best reply for beliefs, respectively, lower than and higher than $\frac{|U_B|}{U_G+|U_B|}$.

ities among these expected utilities, we find that LN with $e^* = 0$ is best reply for DM when $\mu_r \leq \min\left\{\frac{(e^*+(1-p(e^*))\gamma|U_B|)}{(p(e^*)U_G+(1-p(e^*))|U_B|)\gamma}, \frac{|U_B|}{U_G+|U_B|}\right\}$, while LY with $e^* = 0$ is best reply when $\mu_r \geq \max\left\{\frac{(e^*+(1-p(e^*))|U_B|)}{(p(e^*)U_G+(1-p(e^*))|U_B|)}, \frac{|U_B|}{U_G+|U_B|}\right\}$.

Figure 9 summarizes DM's optimal behavior as a function of μ, providing a counterpart of Proposition 1 for the current setup. We note that while the quality of DM's optimal behavior remains substantially unchanged with respect to the model of Section 3, the relaxation of the hypothesis of just two elaboration levels gives an extra role to U_G and U_B: under HYN the optimal elaboration intensity increases, decreases, or is constant in expected quality depending on whether $U_G > |U_B|$, $U_G < |U_B|$, or $U_G = |U_B|$. This is because the nature of the stake – i.e., whether it is most important to get the good offer or to avoid the bad one – determines whether direct information on quality and expected quality are complements or substitutes.

We also note that the optimal choice by P is still described by Proposition 2, meaning that the actual level of e^* under HYN is irrelevant to P. By the same token, Proposition 8 and Proposition 3 remain true. Further, the characterization of equilibria made by Proposition 4 through 7 still holds in qualitative terms, but the statements have to be adjusted by replacing conditions on c_e with the appropriate conditions from those described above.

Finally, we observe that the current extension could be slightly changed by assuming that with probability $p(e)$ the signal σ is correct, as before, but, differently from before, with probability $1 - p(e)$ the signal does not arrive at all. This represents the case where failure to extract from signal σ reliable information on q does not result in extracting wrong information, but just no information. While in this alternative setup the optimal choice of elaboration by DM would be partly different under HYN – potentially u-shaped – the substance of what discussed in this subsection remains true.

8 Conclusions

In this paper we have proposed a model of persuasion that incorporates the insights of social and cognitive psychology on dual process reasoning. We have framed persuasion activities within a sender-receiver model where the sender wants to persuade the receiver to accept his offer. Both agents are rational and Bayesian, but the decision-maker has to pay a cognitive cost to extract all information from the message sent by the persuader. The main novelty of our approach is the consideration of two distinct cognitive processes in the elaboration of information: an initial automatic low elaboration that requires little cognitive effort but entails coarse thinking – i.e., the decision-maker heuristically relies on category-wide information to assess the quality of the offer – and deliberate high elaboration that is more costly in terms of cognitive resources but provides precise information on the quality of the offer. This setup allows us to endow the persuader with a novel strategic tool of persuasion – i.e., reference cues – that we model as references to categories of objects. The proposed model, despite its simplicity, has proved to be rich enough to provide predictions that are in line with well documented findings in the social psychology of persuasion. Also, the model well fits, and hence rationalizes, the outcomes of various persuasion activities in the economic, political, and social spheres.

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A Appendix - Proofs and related technical details

Preliminarily, we denote with $\beta_r(\rho_G, \rho_B)$ the belief on quality conditional on cue $r \in \{x, y\}$ given P's behavior $\rho(G) \in \{x, y\}$, $\rho(B) \in \{x, y\}$. Such beliefs are derived by means of the Bayes rule.

A.1 Beliefs on quality conditional on cue given P's behavior

$$\hat{\beta}_x(\rho(G)=y, \rho(B)=y) = \beta_x \tag{1}$$

$$\hat{\beta}_x(\rho(G)=y, \rho(B)=x) = \frac{\alpha_x \beta_x}{\alpha_x + \alpha_P \alpha_B} \tag{2}$$

$$\hat{\beta}_x(\rho(G)=x, \rho(B)=x) = \frac{\alpha_x \beta_x + \alpha_P \alpha_G}{\alpha_x + \alpha_P} \tag{3}$$

$$\hat{\beta}_x(\rho(G)=x, \rho(B)=y) = \frac{\alpha_x \beta_x + \alpha_P \alpha_G}{\alpha_x + \alpha_P \alpha_G} \tag{4}$$

$$\hat{\beta}_y(\rho(G)=y, \rho(B)=y) = \frac{\alpha_y \beta_y + \alpha_P \alpha_G}{\alpha_y + \alpha_P} \tag{5}$$

$$\hat{\beta}_y(\rho(G)=y, \rho(B)=x) = \frac{\alpha_y \beta_y + \alpha_P \alpha_G}{\alpha_y + \alpha_P \alpha_G} \tag{6}$$

$$\hat{\beta}_y(\rho(G)=x, \rho(B)=x) = \beta_y \tag{7}$$

$$\hat{\beta}_y(\rho(G)=x, \rho(B)=y) = \frac{\alpha_y \beta_y}{\alpha_y + \alpha_P \alpha_B} \tag{8}$$

A.2 Proof of Proposition 2

Proof. Suppose that DM chooses $\delta(x) = \delta(y)$. If $\delta(x) = \delta(y) = LY$ or $\delta(x) = \delta(y) = HYN$ and P is of type G, then P's utility is $V - c_r$. If instead $\delta(x) = \delta(y) = LN$ or $\delta(x) = \delta(y) = HYN$ and P is of type B, then P's utility is $-c_r$. Since $0 = c_y < c_x$, $r = y$ is optimal for P independently of his type.

Suppose that DM chooses $\delta(x) = LY$ and $\delta(y) = LN$. P's utility is equal to $V - c_x > 0$, if $r = x$, and to 0, if $r = y$. Hence, $r = x$ is optimal for P independently of his type.

Suppose that DM chooses $\delta(x) = HYN$ and $\delta(y) = LN$. If P is of type G then his utility is equal to $V - c_x > 0$, if $r = x$, and to 0, if $r = y$. Hence, $r = x$ is optimal for type G. If P is of type B then his utility is equal to $-c_x$, if $r = x$, and to 0, if $r = y$. Since $0 < c_x$, $r = y$ is optimal for type B.

Suppose that DM chooses $\delta(x) = LY$ and $\delta(y) = HYN$. If P is of type G then his utility is equal to $V - c_x$, if $r = x$, and to V, if $r = y$. Since $0 = c_y < c_x$, $r = y$ is optimal for type G. If P is of type B then his utility is equal to $V - c_x > 0$, if $r = x$, and to 0, if $r = y$. Hence, $r = x$ is optimal for type B. □

A.3 Proof of Proposition 3

Preliminarily, it is useful to introduce the parameter χ which measures the degree of coarse thinking, or, to say that in other words, how coarse is coarse thinking. We assume that χ ranges from 0 to ∞, where χ close to 0 means that low elaboration allows in any case a good level of elaboration, making DM uncertain between the current offer by P and only a few other offers, while χ very high means that thinking is very coarse when DM resorts to low elaboration, and hence the number of offers among which she is unable to distinguish is very large. More precisely, if we denote with N_x, N_y, and N_P the absolute number of offers that, respectively, fall into category x and are not made by P, fall into category y and are not made by P, and are made by P, we have that N_x is non-decreasing in χ and $N_x \to \infty$ when $\chi \to \infty$, and an analogous assumption is made for N_y, while we can reasonably assume that N_P does not depend on χ. We also define $\alpha_x = N_x/(N_x + N_y + N_P)$, $\alpha_y = N_y/(N_x + N_y + N_P)$ and $\alpha_P = N_P/(N_x + N_y + N_P)$. We then have that α_P is non-increasing in χ, $\alpha_P \to 0$ when $\chi \to \infty$, and both α_x and α_y are non-decreasing in χ and $\alpha_x + \alpha_y \to 1$ when $\chi \to \infty$.

Proof. We observe that $\hat{\beta}_x(\rho(G), \rho(B))$ converges to β_x and $\hat{\beta}_y(\rho(G), \rho(B))$ converges to β_y irrespective of ρ when the degree of coarse thinking grows larger and larger, i.e., χ tends to infinity. This is evident when looking at (1), (2), (3), (4), (5), (6), (7) and (8).

We assume that β_x and β_y are interior points to the intervals of beliefs that determine DM's best choices according to Proposition 1. More precisely, β_x and β_y are both different from $c_e/\gamma U_G$ and $1 - c_e/\gamma|U_B|$ if $c_e < \gamma U_G|U_B|/\gamma(U_G + |U_B|)$, and different from $|U_B|/(U_G + |U_B|)$ if $c_e \geq U_G|U_B|/\gamma(U_G + |U_B|)$. By so doing we are neglecting values that have measure zero in the parameters space.

We now build a profile that we then check to be an equilibrium. We set $\delta(x)$ and $\delta(y)$ equal to the best action by DM against a belief equal to β_x and β_y, respectively, as show by Proposition 1. We set $\rho(G)$ and $\rho(B)$ equal to the best action by P conditional on G and B, respectively, against $\delta(x)$ and $\delta(y)$, as shown by Proposition 2.

By construction, in the above profile P is best replying to δ, while DM is best replying given β_x and β_y, which are not equilibrium beliefs. However, we can choose χ high enough that, whatever ρ is chosen by P, $\hat{\beta}_x(\rho(G), \rho(B))$ and $\hat{\beta}_y(\rho(G), \rho(B))$ are very close to β_x and β_y, respectively. This means that DM is best replying even against $\hat{\beta}_x(\rho(G), \rho(B))$ and $\hat{\beta}_y(\rho(G), \rho(B))$, since β_x and β_y are interior points to the intervals of beliefs that determine DM's best choices.

We have just proved equilibrium existence. To understand that such equilibrium is unique, we simply observe that, when χ is large enough, the best reply by DM is uniquely determined whatever strategy is chosen by P, and P's best reply against such an optimal behavior by DM is uniquely determined as well. □

B Persuasion equilibria

Preliminarily, let $\delta : \{x, y\} \to \{LY, HYN, LN\}$ be the function describing DM's behavior conditional upon the observation of the reference cue.

B.1 Pooling equilibria: High and low signals

We say that an equilibrium is pooling with high signal if $\rho(G) = \rho(B) = x$. Note that the rejection of any offer associated with cue y, i.e., $\delta(y) = LN$, and the acceptance of any offer associated with cue x, i.e., $\delta(x) = LY$, is the only behavior by DM that can sustain a pooling equilibrium with high signal. Indeed, in such a case P can have his offer accepted only by using the reference cue x, and this independently of whether he is of type G or of type B. We also note that a pooling equilibrium with high signal can exist for low elaboration costs – so that HYN is optimal for intermediate values of expected quality – and for high elaboration costs – so high that HYN is never optimal. The following proposition provides the conditions for the existence of a pooling equilibrium with high signal.

Proposition **4** (Pooling equilibrium with high signal). *The profile (ρ, δ) such that $\rho(G) = \rho(B) = x$, $\delta(x) = LY$, $\delta(y) = LN$ is an equilibrium if and only if:*

(4.1) $\hat{\beta}_x(\rho(G), \rho(B)) \geq 1 - \frac{c_e}{\gamma|U_B|}$ *and* $\hat{\beta}_x(\rho(G), \rho(B)) \geq \frac{|U_B|}{U_G + |U_B|}$*;*

(4.2) $\hat{\beta}_y(\rho(G), \rho(B)) \leq \frac{c_e}{\gamma U_G}$ *and* $\hat{\beta}_y(\rho(G), \rho(B)) \leq \frac{|U_B|}{U_G + |U_B|}$*.*

There is no other equilibrium profile (ρ, δ) such that $\rho(G) = \rho(B) = x$, whatever the values of $\hat{\beta}_x(\rho(G), \rho(B))$ and $\hat{\beta}_y(\rho(G), \rho(B))$.

Proof. The last claim of the proposition follows directly from Proposition 2: for $r = x$ to be P's optimal choice independently of his type, DM's choice must be such that $\delta(x) = LY$ and $\delta(y) = LN$.

So, let (ρ, δ) be an equilibrium. Note that, along the equilibrium path, $\mu = \hat{\beta}_x(\rho(G), \rho(B))$ if DM sees $r = x$ and $\mu = \hat{\beta}_y(\rho(G), \rho(B))$ if DM sees $r = y$. Hence, from Proposition 1 follows that 4.1 must hold for DM to find $\delta(x) = LY$ optimal and that 4.2 must hold for DM to find $\delta(y) = LN$ optimal.

Suppose now that 4.1 and 4.2 hold. Then, from Proposition 1 follows that $\delta(x) = LY$ and $\delta(y) = LN$ is optimal for DM. Hence, by Proposition 2 we can conclude that the profile (ρ, δ) is an equilibrium. □

We say that an equilibrium is pooling with low signal if $\rho(G) = \rho(B) = y$. The existence of such an equilibrium depends on the convenience for DM to behave in the same way when observing x or y, i.e., $\delta(x) = \delta(y)$. Indeed, when this occurs, P clearly finds it optimal to choose y irrespectively of whether he is of type G or of type B, since DM's behavior is not affected by the choice of the reference cue, and y costs less than x. We note that there are variants of this type of equilibrium, depending on the behavior held by DM. If elaboration costs are so large that HYN is never optimal then there are two cases: either $\delta(x) = \delta(y) = LN$ or $\delta(x) = \delta(y) = LY$. Otherwise, if elaboration costs are not so large, then there is also a third possibility, namely that $\delta(x) = \delta(y) = HYN$. The following proposition provides the conditions for the existence of a pooling equilibrium with low signal for each of the three variants.

PROPOSITION **5** (Pooling equilibrium with low signal).

5.1 The profile (ρ, δ) such that $\rho(G) = \rho(B) = y$, $\delta(x) = \delta(y) = LN$ is an equilibrium if and only if:

5.1.1 $\hat{\beta}_x(\rho(G), \rho(B)) \leq \frac{c_e}{\gamma U_G}$ *and* $\hat{\beta}_x(\rho(G), \rho(B)) \leq \frac{|U_B|}{U_G + |U_B|}$*;*

5.1.2 $\hat{\beta}_y(\rho(G), \rho(B)) \leq \frac{c_e}{\gamma U_G}$ *and* $\hat{\beta}_y(\rho(G), \rho(B)) \leq \frac{|U_B|}{U_G + |U_B|}$.

5.2 The profile (ρ, δ) such that $\rho(G) = \rho(B) = y$, $\delta(x) = \delta(y) = HYN$ is an equilibrium if and only if:

5.2.1 $\hat{\beta}_x(\rho(G), \rho(B)) \geq \frac{c_e}{\gamma U_G}$ *and* $\hat{\beta}_x(\rho(G), \rho(B)) \leq 1 - \frac{c_e}{|\gamma U_B|}$*;*

5.2.2 $\hat{\beta}_y(\rho(G), \rho(B)) \geq \frac{c_e}{\gamma U_G}$ *and* $\hat{\beta}_y(\rho(G), \rho(B)) \leq 1 - \frac{c_e}{|\gamma U_B|}$.

5.3 The profile (ρ, δ) such that $\rho(G) = \rho(B) = y$, $\delta(x) = \delta(y) = LY$ is an equilibrium if and only if:

5.3.1 $\hat{\beta}_x(\rho(G), \rho(B)) \geq 1 - \frac{c_e}{|\gamma U_B|}$ *and* $\hat{\beta}_x(\rho(G), \rho(B)) \geq \frac{|U_B|}{U_G + |U_B|}$*;*

5.3.2 $\hat{\beta}_y(\rho(G), \rho(B)) \geq 1 - \frac{c_e}{|\gamma U_B|}$ *and* $\hat{\beta}_y(\rho(G), \rho(B)) \geq \frac{|U_B|}{U_G + |U_B|}$.

There is no other equilibrium profile (ρ, δ) such that $\rho(G) = \rho(B) = y$, whatever the values of $\hat{\beta}_x(\rho(G), \rho(B))$ and $\hat{\beta}_y(\rho(G), \rho(B))$.

Proof. The last claim of the proposition follows directly from Proposition 2: for $r = y$ to be P's optimal choice independently of his type, DM's choice must be such that $\delta(x) = \delta(y)$.

Suppose (ρ, δ) of 5.1 be an equilibrium. Note that, along the equilibrium path, $\mu = \hat{\beta}_x(\rho(G), \rho(B))$ if DM sees $r = x$ and $\mu = \hat{\beta}_y(\rho(G), \rho(B))$ if DM sees $r = y$. Hence, from Proposition 1 follows that 5.1.1 and 5.1.2 must hold for to find $\delta(x) = \delta(y) = LN$ optimal.

Suppose now that 5.1.1 and 5.1.2. Then, from Proposition 1 follows that $\delta(x) = \delta(y) = LN$ is optimal for DM. Hence, by Proposition 2 we can conclude that the profile (ρ, δ) is an equilibrium.

Claims 5.2 and 5.3 can be proved with analogous arguments. □

B.2 Separating equilibrium: High quality going with high signal

We say that an equilibrium is separating with high quality going with high signal if $\rho(G) = x$ and $\rho(B) = y$. Note that this behavior by P can form an equilibrium only if DM chooses $\delta(x) = HYN$ and $\delta(y) = LN$. In such a case, the persuader of type G finds it optimal to incur the cost of sending x, since this leads his offer to be accepted, while the persuader of type B prefers to save on costs and send reference cue y, since in no case his offer will be accepted. Note also that for this equilibrium to exist elaboration costs must be low enough so that DM actually best replies with HYN for intermediate values of μ.

PROPOSITION **6** (Separating equilibrium with signaling).
The profile (ρ, δ) such that $\rho(G) = x$, $\rho(B) = y$, $\delta(x) = HYN$, $\delta(y) = LN$ is an equilibrium if and only if:

6.1 $\hat{\beta}_x(\rho(G), \rho(B)) \geq \frac{c_e}{\gamma U_G}$ *and* $\hat{\beta}_x(\rho(G), \rho(B)) \leq 1 - \frac{c_e}{|\gamma U_B|}$;

6.2 $\hat{\beta}_y(\rho(G), \rho(B)) \leq \frac{c_e}{\gamma U_G}$ *and* $\hat{\beta}_y(\rho(G), \rho(B)) \leq \frac{|U_B|}{U_G + |U_B|}$.

Proof. Let (ρ, δ) be an equilibrium. Note that, along the equilibrium path, $\mu = \hat{\beta}_x(\rho(G), \rho(B))$ if DM sees $r = x$ and $\mu = \hat{\beta}_y(\rho(G), \rho(B))$ if DM sees $r = y$. Hence, from Proposition 1 follows that 6.1 must hold for DM to find $\delta(x) = HYN$ optimal and that 6.2 must hold for DM to find $\delta(y) = LN$ optimal.

Suppose now that 6.1 and 6.2 hold. Then, from Proposition 1 follows that $\delta(x) = HYN$ and $\delta(y) = LN$ is an optimal choice for DM. Hence, by Proposition 2 we can conclude that the profile (ρ, δ) is an equilibrium. □

B.3 Separating equilibrium: High quality going with low signal

We say that an equilibrium is separating with high quality going with low signal if $\rho(G) = y$ and $\rho(B) = x$. Note that this behavior by P can form an equilibrium only if DM chooses $\delta(y) = HYN$ and $\delta(x) = LY$. Indeed, in such a case the persuader of type G finds it optimal to save on costs and send y, since this leads nevertheless his offer to be accepted, while the persuader of type B finds it optimal to pay the cost of sending x, since this is the only way to have his offer accepted. Note also that, as for the other separating equilibrium, in order for this equilibrium to exist elaboration costs must be low enough so that DM actually best replies with HYN for intermediate values of μ.

PROPOSITION **7** (Separating equilibrium with reverse-signaling).
The profile (ρ, δ) such that $\rho(G) = y$, $\rho(B) = x$, $\delta(x) = LY$, $\delta(y) = HYN$ is an equilibrium if and only if:

7.1 $\hat{\beta}_x(\rho(G), \rho(B)) \geq 1 - \frac{c_e}{\gamma |U_B|}$ *and* $\hat{\beta}_x(\rho(G), \rho(B)) \geq \frac{|U_B|}{U_G + |U_B|}$;

7.2 $\hat{\beta}_y(\rho(G), \rho(B)) \geq \frac{c_e}{\gamma U_G}$ *and* $\hat{\beta}_y(\rho(G), \rho(B)) \leq 1 - \frac{c_e}{\gamma |U_B|}$.

Proof. Let (ρ, δ) be an equilibrium. Note that, along the equilibrium path, $\mu = \hat{\beta}_x(\rho(G), \rho(B))$ if DM sees $r = x$ and $\mu = \hat{\beta}_y(\rho(G), \rho(B))$ if DM sees $r = y$. Hence, from Proposition 1 follows that 7.1 must hold for DM to find $\delta(x) = LY$ optimal and that 7.2 must hold for DM to find $\delta(y) = HYN$ optimal.

Suppose now that 7.1 and 7.2 hold. Then, from Proposition 1 follows that $\delta(x) = LY$ and $\delta(y) = HYN$ is an optimal choice for DM. Hence, by Proposition 2 we can conclude that the profile (ρ, δ) is an equilibrium. □

C Appendix - Multiplicity and existence of persuasion equilibria: Examples and results

C.1 Multiple equilibria can coexist

An example of multiplicity of equilibria is depicted in Figure 10, where the following two equilibria coexist: a pooling equilibrium with high signal and a separating equilibrium.

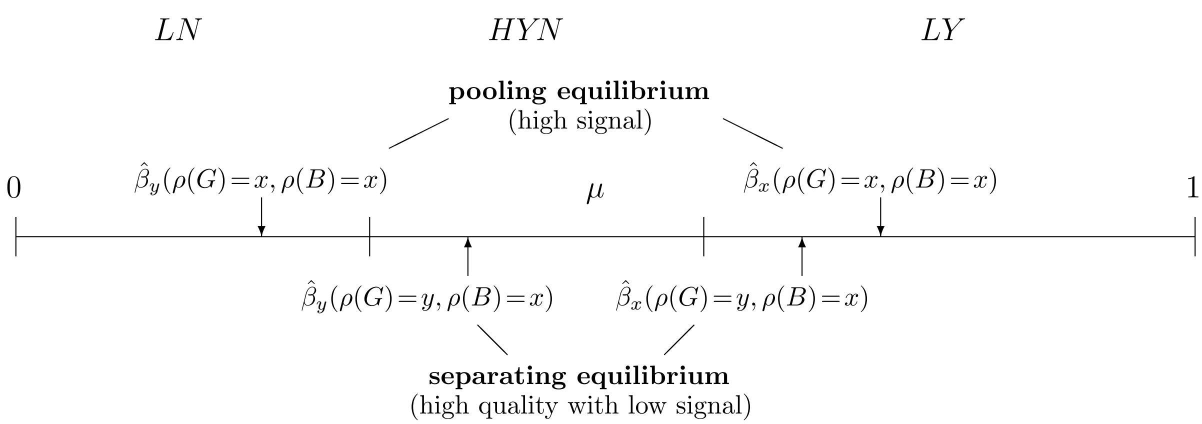

Figure 10: An example of the coexistence of two equilibria, for $c_e < \frac{U_G|U_B|}{(\gamma(U_G+|U_B|)}$.

Looking at Figure 10 we see that, when both types of P choose cue x, then the expected quality $\hat{\beta}_x$ is high and DM's best reply is LY, while the expected quality $\hat{\beta}_y$ is so low that DM's best reply is LY; this justifies the pooling equilibrium where both type G and type B choose the high signal x. At the same time, if type G switches from x to y, it may happen that $\hat{\beta}_x$ remain high enough to have that DM best replies with LY, and $\hat{\beta}_y$ raises entering the region where DM best replies with HYN; this is what occurs in the case represented in the figure, and it justifies the reverse-signaling equilibrium.

We note, however, that not all types of equilibria can coexist. The following proposition lists the possible cases of coexistence of equilibria:

Proposition **8** (Equilibrium multiplicity and coexistence)**.**
Multiple equilibria can exist, but only in the following pairs:

- *a pooling where* $\rho(G)=\rho(B)=x$ *and a separating where* $\rho(G)=y$ *and* $\rho(B)=x$*;*
- *a pooling where* $\rho(G)=\rho(B)=y$ *and a separating where* $\rho(G)=x$ *and* $\rho(B)=y$*;*

- *a pooling where $\rho(G)=\rho(B)=x$ and a pooling where $\rho(G)=\rho(B)=y$.*

The proof of Proposition 8 can be found in Subsection C.3. Here we provide the intuition why a reverse-signaling equilibrium – where high quality goes with low signal – cannot coexist with a signaling equilibrium – where high quality goes with high signal. In a signaling equilibrium type G sends cue x and type B sends cue y, and for B to send cue y DM's belief conditional on cue x must be low enough not to induce DM to play LY – otherwise type B would find it profitable to deviate from sending cue y to sending cue x. On the contrary, in a reverse-signaling equilibrium type G sends cue y and type B sends cue x, but for B to send cue x DM's belief conditional on cue x must be high enough to have DM play LY – so that B actually has the offer accepted if he sends cue x. This two conditions are incompatible because having G sending y and B sending x decreases the belief conditional on x with respect to having G sending x and B sending y, so that if beliefs are low enough to sustain a signaling equilibrium then they cannot be high enough to sustain a reverse-signaling equilibrium.

C.2 An equilibrium may fail to exist

Another possibile occurrence is that no equilibrium exists in pure strategies. An example of equilibrium inexistence is depicted in Figure 11, where it can be easily checked that for any given behavior by P the best reply by DM is such that at least one type of persuader strictly gains by deviating.

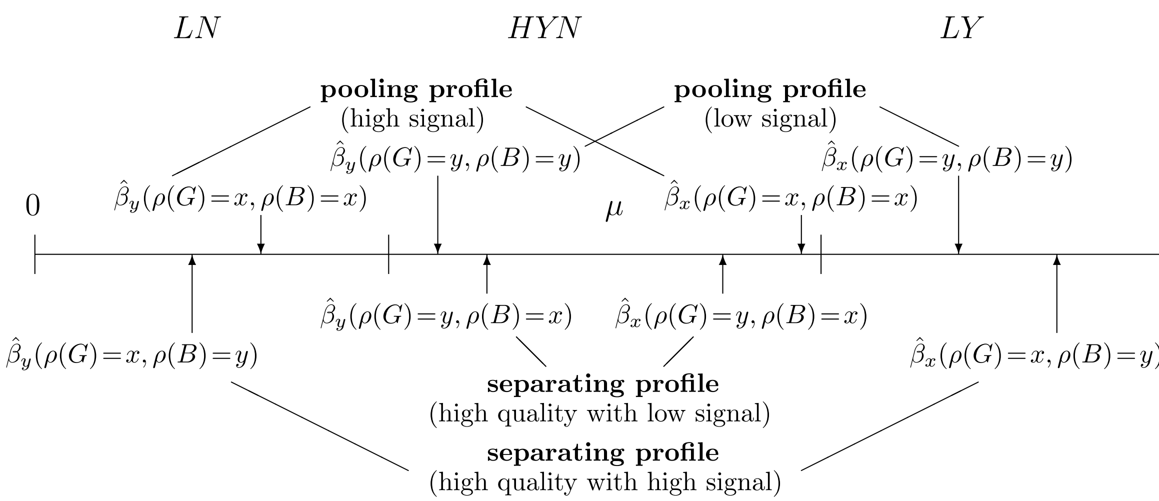

Figure 11: An example where no equilibrium exists, for $c_e < \frac{U_G|U_B|}{\gamma(U_G+|U_B|)}$.

Looking at Figure 11 we see that both the separating profile where high quality goes with low signal and the pooling profile with high signal cannot be equilibria because DM best replies with

HYN to cue x; in such a case, indeed, the persuader of type B would prefer to send cue y and save on costs, since he will never see his offer accepted. Similarly, both the pooling profile with low signal and the separating profile with high quality going with high signal cannot be equilibria as well, because DM best replies with LY to cue x, and hence the persuader of type B would prefer to send cue x (so to have his offer accepted) instead of y (being his offer rejected with such a cue). We remark again that the example depicted in the figure – and more in general the possibility that no equilibrium exists – is due to the fact that $\hat{\beta}_x$ and $\hat{\beta}_y$ move along the segment as types G and B change the choice of cues, thus changing also the optimal behavior of DM.

Actually, an equilibrium exists if mixed strategies are considered. To be convinced of this, start considering the pooling profile where both G and B choose signal x; as already remarked, this is not an equilibrium because type B has a profitable deviation from x to y. Let us now suppose that type B chooses a mixed strategy where the probability of playing y progressively increases starting from zero. Consequently, $\hat{\beta}_y$ decreases and hence remains in the region where DM best replies with LN, while $\hat{\beta}_x$ progressively raises until it reaches the point where DM is indifferent between HYN and LY, and hence she can optimally randomize between HYN and LY. In particular, she can randomize with probabilities that make type B indifferent between x and y. We have hence found a mixed strategy equilibrium where type G chooses x, type B randomizes between y and x, and DM chooses LN if y and randomizes between HYN and LY if x. Starting from the pooling equilibrium where both G and B choose signal y, and following a similar reasoning, we can find another mixed strategy equilibrium with the main difference that type G chooses y and DM chooses HYN if y.

C.3 Proof of Proposition 8

Preliminarily, we give the following straightforward results which will be used in the subsequent proof. For α_x, α_y, α_G, β_x, and β_y strictly comprised between 0 and 1, the following inequalities necessarily hold:

$$\hat{\beta}_x(\rho(G)=y, \rho(B)=x) < \hat{\beta}_x(\rho(G)=y, \rho(B)=y) \tag{9}$$

$$\hat{\beta}_x(\rho(G)=y, \rho(B)=x) < \hat{\beta}_x(\rho(G)=x, \rho(B)=x) \tag{10}$$

$$\hat{\beta}_x(\rho(G)=y, \rho(B)=y) < \hat{\beta}_x(\rho(G)=x, \rho(B)=y) \tag{11}$$

$$\hat{\beta}_x(\rho(G)=x, \rho(B)=x) < \hat{\beta}_x(\rho(G)=x, \rho(B)=y) \tag{12}$$

$$\hat{\beta}_y(\rho(G)=x, \rho(B)=y) < \hat{\beta}_y(\rho(G)=x, \rho(B)=x) \tag{13}$$

$$\hat{\beta}_y(\rho(G)=x, \rho(B)=y) < \hat{\beta}_y(\rho(G)=y, \rho(B)=y) \tag{14}$$

$$\hat{\beta}_y(\rho(G)=y, \rho(B)=y) < \hat{\beta}_y(\rho(G)=y, \rho(B)=x) \tag{15}$$

$$\hat{\beta}_y(\rho(G)=x, \rho(B)=x) < \hat{\beta}_y(\rho(G)=y, \rho(B)=x) \tag{16}$$

The following is a check that the above inequalities indeed hold:

$$\hat{\beta}_x(\rho(G)=y,\rho(B)=x)=\frac{\alpha_x\beta_x}{\alpha_x+\alpha_P\alpha_B}=\frac{\beta_x}{1+1-\frac{\alpha_P\alpha_B}{\alpha_x}}<\beta_x=\hat{\beta}_x(\rho(G)=y,\rho(B)=y) \tag{17}$$

$$\begin{aligned}\hat{\beta}_x(\rho(G)=y,\rho(B)=x)&=\frac{\alpha_x\beta_x}{\alpha_x+\alpha_P\alpha_B}<\frac{\alpha_x\beta_x+\alpha_P\alpha_G}{\alpha_x+\alpha_P\alpha_B+\alpha_P\alpha_G}=\\&=\frac{\alpha_x\beta_x+\alpha_P\alpha_G}{\alpha_x+\alpha_P}=\hat{\beta}_x(\rho(G)=x,\rho(B)=x)\end{aligned} \tag{18}$$

$$\begin{aligned}\hat{\beta}_x(\rho(G)=y,\rho(B)=y)&=\beta_x=\beta_x\left(\frac{\alpha_x+\alpha_P\alpha_G}{\alpha_x+\alpha_P\alpha_G}\right)=\frac{\alpha_x\beta_x+\alpha_P\alpha_G\beta_x}{\alpha_x+\alpha_P\alpha_G}<\\&<\frac{\alpha_x\beta_x+\alpha_P\alpha_G}{\alpha_x+\alpha_P\alpha_G}=\hat{\beta}_x(\rho(G)=x,\rho(B)=y)\end{aligned} \tag{19}$$

$$\begin{aligned}\hat{\beta}_x(\rho(G)=x,\rho(B)=x)&=\frac{\alpha_x\beta_x+\alpha_P\alpha_G}{\alpha_x+\alpha_P}=\frac{\alpha_x\beta_x+\alpha_P\alpha_G}{\alpha_x+\alpha_P\alpha_G+\alpha_P\alpha_B}<\\&<\frac{\alpha_x\beta_x+\alpha_P\alpha_G}{\alpha_x+\alpha_P\alpha_G}=\hat{\beta}_x(\rho(G)=x,\rho(B)=y)\end{aligned} \tag{20}$$

$$\hat{\beta}_y(\rho(G)=x,\rho(B)=y)=\frac{\alpha_y\beta_y}{\alpha_y+\alpha_P\alpha_B}=\frac{\beta_y}{1+\frac{\alpha_P\alpha_B}{\alpha_y}}<\beta_y=\hat{\beta}_y(\rho(G)=x,\rho(B)=x) \tag{21}$$

$$\begin{aligned}\hat{\beta}_y(\rho(G)=x,\rho(B)=y)&=\frac{\alpha_y\beta_y}{\alpha_y+\alpha_P\alpha_B}<\frac{\alpha_y\beta_y+\alpha_P\alpha_G}{\alpha_y+\alpha_P\alpha_B+\alpha_P\alpha_G}=\\&=\frac{\alpha_y\beta_y+\alpha_P\alpha_G}{\alpha_y+\alpha_P}=\hat{\beta}_y(\rho(G)=y,\rho(B)=y)\end{aligned} \tag{22}$$

$$\begin{aligned}\hat{\beta}_y(\rho(G)=y,\rho(B)=y)&=\frac{\alpha_y\beta_y+\alpha_P\alpha_G}{\alpha_y+\alpha_P}=\frac{\alpha_y\beta_y+\alpha_P\alpha_G}{\alpha_y+\alpha_P\alpha_G+\alpha_P\alpha_B}\\&<\frac{\alpha_y\beta_y+\alpha_P\alpha_G}{\alpha_y+\alpha_P\alpha_G}=\hat{\beta}_y(\rho(G)=y,\rho(B)=x)\end{aligned} \tag{23}$$

$$\begin{aligned}\hat{\beta}_y(\rho(G)=x,\rho(B)=x)&=\beta_y=\beta_y\left(\frac{\alpha_y+\alpha_P\alpha_G}{\alpha_y+\alpha_P\alpha_G}\right)=\frac{\alpha_y\beta_y+\alpha_P\alpha_G\beta_y}{\alpha_y+\alpha_P\alpha_G}<\\&<\frac{\alpha_y\beta_y+\alpha_P\alpha_G}{\alpha_y+\alpha_P\alpha_G}=\hat{\beta}_y(\rho(G)=y,\rho(B)=x)\end{aligned} \tag{24}$$

We note that inequality (17) follows from $(\alpha_P\alpha_B)/\alpha_x > 0$, inequality (18) from $\alpha_P\alpha_G > 0$ and (2) being strictly lower than 1, inequalities (19) and (24) from $\alpha_P\alpha_G\beta_x < \alpha_P\alpha_G$, inequalities (20) and (23) from $\alpha_P\alpha_B > 0$, inequality (21) from $(\alpha_P\alpha_B)/\alpha_y > 0$, and inequality (22) from $\alpha_P\alpha_G > 0$ and (8) being strictly lower than 1.

Proof. From (9) follows that condition 7.1 is incompatible with conditions 5.1.1 and 5.2.1. Moreover, from (15) follows that condition 7.2 is incompatible with condition 5.3.2. This proves that a separating equilibrium where $\rho(G) = y$ and $\rho(B) = x$ and a pooling equilibrium where $\rho(G) = \rho(B) = y$ cannot coexist.

From (12) follows that condition 6.1 is incompatible with condition 4.1. This proves that a separating equilibrium where $\rho(G) = x$ and $\rho(B) = y$ and a pooling equilibrium where $\rho(G) = \rho(B) = x$ cannot coexist.

From (9) and (11) follows that $\hat{\beta}_x(\rho(G) = y, \rho(B) = x) < \hat{\beta}_x(\rho(G) = x, \rho(B) = y)$, which in turn implies that condition 7.1 is incompatible with condition 6.1. This proves that the two types of separating equilibria cannot coexist.

To see that the two types of pooling can coexists suppose that $|U_B|/(U_G + |U_B|) < 1$. If β_x is close enough to 1 so that 5.3.1 is satisfied and that β_y is close enough to 0 so that condition 4.2 is satisfied. For α_G large enough, also 4.1 is satisfied. To have also 5.3.2 satisfied it is enough to have α_P and α_G sufficiently large.

To see that a pooling equilibrium where $\rho(G) = \rho(B) = x$ and a separating equilibrium where $\rho(G) = y$ and $\rho(B) = x$ can coexist, note that from $\hat{\beta}_x(\rho(G) = y, \rho(B) = x) < \hat{\beta}_x(\rho(G) = x, \rho(B) = y)$ follows that condition 7.1 implies condition 4.1. Moreover, by (16) we can set c_e, U_G and U_B such that $U_G\hat{\beta}_y(\rho(G) = x, \rho(B) = x) \leq c_e \leq U_G\hat{\beta}_y(\rho(G) = y, \rho(B) = x)$, and $\hat{\beta}_y(\rho(G) = x, \rho(B) = x) \leq |U_B|/(U_G + |U_B|) \leq \hat{\beta}_y(\rho(G) = y, \rho(B) = x)$, so that both 7.2 and 4.2 are satisfied.

A similar argument can be applied to show that a pooling equilibrium where $\rho(G) = \rho(B) = y$ and a separating equilibrium where $\rho(G) = x$ and $\rho(B) = y$ can coexist. □

D Appendix - Model extensions

D.1 Many offer qualities

Consider the case where the quality of offers is not limited to two levels, G and B, but can take many possible values. Let us index qualities on the interval of the real line $[\underline{q}, \overline{q}]$, where $\underline{q} > 0$ is minimum quality and $\overline{q}$ is maximum quality. The quality of the offers not coming from P is determined according to the cumulative distribution F^N in case N is chosen and according to the cumulative distribution F^P in case P is chosen. The values of βs are modified accordingly.

DM's utility is given by $U(q)$, which is strictly increasing in q and takes both positive and negative values over $[\underline{q}, \overline{q}]$. In particular, there exists $\tilde{q}$ such that $U(\tilde{q}) = 0$. DM would like to accept any offer of quality $q \geq \tilde{q}$ and reject any offer of quality $q \leq \tilde{q}$. Hence, optimal choice by DM is still described by Proposition 1, where U_G and U_B are replaced by, respectively, $\widetilde{U}_G = \int_{\tilde{q}}^{\overline{q}} U(q)\mathrm{d}F(q)$ and $\widetilde{U}_B = \int_{\underline{q}}^{\tilde{q}} U(q)\mathrm{d}F(q)$, with $F = \alpha_P F^P + (1 - \alpha_P)F^N$.

Further, persuaders of types $q \geq \tilde{q}$ find it optimal to behave like G in the model of Section 3, while persuaders of types $q \leq \tilde{q}$ find it optimal to behave like B. So, Proposition 2 still describes the optimal choice by P conditionally on the potentially optimal behavior by DM, where however $\rho(G)$ and $\rho(B)$ are interpreted as referring to, respectively, types in $[\tilde{q}, \overline{q}]$ and types in $[\underline{q}, \tilde{q}]$, and where again U_G and U_B are replaced by $\widetilde{U}_G$ and $\widetilde{U}_B$.

As a consequence, the substance of the findings reported in Proposition 4 through 3 remains true when we allow for many different qualities of the offer.

D.2 Many offer categories and reference cues

Consider the case where the categories of objects known by DM, and to which the offer might be referred to, are not just x and y, but possibly a large number. Let Z be the set of natural numbers $\{1, 2, \ldots, n\}$ indexing the different offer categories, with $n \geq 2$ and $z \in Z$ denoting the generic reference cue. Suppose also that both β_z, which denotes the average quality for category z, and c_z, which denotes the cost of sending a cue referring to category z, are strictly increasing in z. Finally, to rule out uninteresting cases let also $c_z < V$ for all $z \in Z$.

We note that the optimal choice by DM is still described by Proposition 1. However, to describe the optimal choice by P conditionally on the potentially optimal behavior by DM one needs to generalize Proposition 2 to the case of many offer categories. We do not enter the details of this, since the forces driving the choice by P remain the same. Indeed, P will choose a reference cue that leads his offer to be accepted, if such a cue exists. We note that, as in the basic setup, type G of P has more chances to attain this objective, since for him it is enough to choose a cue that induces HYN as best reply by DM, while type B of P needs a cue that leads to immediate acceptance – i.e., that leads DM to choose LY. Moreover, among cues that lead to the same best reply by DM, P will choose the one with the minimum index, since that cue is the least costly. The difference with respect to the setup of Section 3 is the larger number of choices that are available to P, and consequently the larger number of strategies for DM, since a strategy for her is now $\delta : Z \to \{LN, HYN, LY\}$. A proposition describing the best reply behavior by P should consider all possible behaviors by DM, and hence it would consist of many cases, whose explicit listing would add little to intuition.

From the detailed description of the potentially optimal choices by DM and P – i.e., the counterparts of Propositions 1 and 2 in this setup – one can obtain results that are in line with Proposition 4 through 7. To see that pooling and separating equilibria can emerge with many categories as well, it is enough to think of the equilibria described for the model of Section 3 and add some further categories whose average quality induces the same best reply by DM as done by x or y, but with the associated reference cues being more costly, so that the additional categories are never chosen by P. Moreover, it is easy to understand that as coarse thinking becomes stronger and stronger, existence and uniqueness of the equilibrium are almost always ensured, similarly to what happens in with Proposition 3.

Even if all types of equilibria remain possible in the presence of more than two categories, we remark that separation with low types sending the a high reference cue becomes the more likely outcome as the number of categories increases and average qualities are more spread all over $[0, 1]$. To show this formally, let us introduce a measure of how categories are densely distributed in terms of their average qualities. More precisely, given $0 \leq \beta_1 < \beta_2 < \ldots < \beta_n \leq 1$, we define ξ to be equal to the largest difference of two consecutive numbers in the above sequence. In other words, ξ is the minimum length that an interval of average qualities has to have to be sure that it contains at least the average quality of one category in Z. So, the lower ξ is, the more densely distributed average qualities are.

We are now ready to state the following result:

Proposition **9** (Separation with many offer categories)**.**
Suppose that $c_e < U_G|U_B|/\gamma(U_G + |U_B|)$ *and* $\xi < \min\{1 - c_e/\gamma|U_B| - c_e/\gamma U_G, c_e/gamma|U_B|\}$*. If coarse thinking is strong enough, then almost always there exists a profile* (δ, ρ) *that is the unique equilibrium and such that* $\hat{\beta}_{\rho(G)} < \hat{\beta}_{\rho(B)}$*,* $\delta(\rho(G)) = HYN$*, and* $\delta(\rho(B)) = LY$*.*

Proof. Since $c_e < U_G|U_B|/\gamma(U_G+|U_B|)$, then there exists an interval of beliefs against which HYN is the best reply by DM. Moreover, since $\xi < \min\{1 - c_e/\gamma|U_B| - c_e/\gamma U_G, c_e/\gamma|U_B|\}$, we are sure that there exist at least one category whose average quality induces HYN as best reply, and at least one category whose average quality induces LY as best reply. We denote with z^{HYN}_{min} the category with the minimum index among those which are best replied with HYN, and we define z^{LY}_{min} analogously. We suppose that z^{HYN}_{min} and z^{HYN}_{min} are interior points in the intervals of beliefs that are best replied, respectively, with HYN and LY. By so doing we are neglecting values that have measure zero in the parameters space.

We now build a profile that we then check to be an equilibrium. For every $z \in Z$, we set $\rho(z)$ as the best action by DM against β_z, as shown by Proposition 1. Then we set $\delta(G) = z_{min}^{HYN}$ and $\delta(B) = z_{min}^{LY}$. We note that such δ selects for both G and B the least costly cue that allows them to have their offer accepted by DM. This shows the optimality of P's strategy against ρ. To understand that ρ is optimal against δ, it is enough to observe that, when coarse thinking is strong enough, $\hat{\beta}_{z_{min}^{HYN}}$ and $\hat{\beta}_{z_{min}^{LY}}$ are arbitrarily close to, respectively, $\beta_{z_{min}^{HYN}}$ and $\beta_{z_{min}^{LY}}$, and hence induce the same best action by DM, since $\beta_{z_{min}^{HYN}}$ and $\beta_{z_{min}^{LY}}$ are interior points in the intervals of best reply behavior by DM.

Finally, uniqueness follows exactly for the same argument shown in Proposition 3, which hence we do not repeat. □

D.3 Cues with fully endogenous quality

Proposition 7 tells us that a reverse-signaling equilibrium can arise in our model, and Proposition 9 states that this type of equilibrium is the only type that is possible when there are many categories and average qualities are sufficiently densely distributed. The combination of these results suggests that reverse-signaling may be a quite relevant case. However, the assumed partial exogeneity of expected average quality that is associated with each reference cue can raise doubts about the relevance of reverse-signaling. Indeed, in the model of Section 3 (and similarly in the model of Subsection D.2) the average quality of offers not coming from P, i.e., β_x and β_y, is exogenous and, in particular, one category of offers is of better average quality than the other by assumption, i.e., $\beta_x > \beta_y$. We observe that in a reverse-signaling equilibrium the average quality remains higher in category x than in category y, despite the fact that when P is called to play he chooses cue y if his type is G and cue x if his type is B. In order for this to occur, the decisions influencing the quality of offers in category x and y that are not made by P should be sustained by sound explanations. In the following we explore one particular explanation: for a subset of offers in each category, cues x and y denote characteristics which provide an intrinsic utility to DM, so that for such offers the choice between x and y directly affects the quality of the offer.

Consider the model presented in Section 3, but modified as follows. Initially, either an offerer O or a persuader P are randomly selected with probability α_O and $\alpha_P = 1 - \alpha_O$, respectively. If P is chosen then the game unfolds as in the original model. Instead, if O is chosen then the game unfolds as follows. Firstly, a type for O is drawn, either the type A – who is endowed with advanced technology – or the type S – who is endowed with standard technology; types are selected with probability α_A and α_S, respectively. Secondly, O chooses between x and y. Lastly, DM observes the reference cue without knowing whether the offer comes from O or from P, and has to decide on both elaboration level and reaction.

The two types of player O are characterized by different technologies that induce different costs to choose x. In particular, the advanced technology allows type A to incur a lower cost than type

S to employ x, i.e., $c_x^A < c_x^S$. We also assume that $V - c_x^A > 0$ and $V - c_x^S < 0$, meaning that type A prefers x, while type S prefers y.[17] Moreover, the offer made by O is such that the cue has an intrinsic value for DM, meaning that x qualifies the offer as good, while y qualifies the offer as bad. In other words, the offer is of good quality if x is chosen, while it is of bad quality if y is chosen. Finally, a strategy by O – which we denote by ω – is a choice between x and y as a function of the type, i.e., $\omega : \{A, S\} \to \{x, y\}$.

[17] We remark that the null payoff that type S obtains by choosing Y if DM replies with N should not be interpreted as "getting nothing", since payoff levels are normalized (see Subsection 6) and, hence, such an occurrence may represent the fact that DM buys at a lower price or a smaller quantity.